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HOPF BIFURCATION ANALYSIS ON A VIRAL DISEASE MODEL WITH SATURATED CONTACT RATE

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Abstract. In this paper, a class of viral disease model with saturated contact rate is considered. By choosing the delay as bifurcation parameter and analyzing the associated characteristic equation, the existence of bifurcation parameter point is determined. If the parameter τ is chosen as a bifurcation parameter, it is found that Hopf bifurcation occurs when the parameter τ passes through a series of critical values. The direction and the stability of Hopf bifurcation periodic solutions are determined by using the normal form theory and the center manifold theorem due to Faria and Maglhalães [20]. Some numerical simulation for justifying the theoretical analysis are also provided. Finally, biological explanations and main conclusions are given.

Keywords. Viral disease model; Delay; Stability; Hopf bifurcation; Periodic solution. **AMS (MOS) subject classification.** 34K20; 34C25.

1 Introduction

The dynamical behaviors of viral models have long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. It is known that there has been considerable interest over the past decade in investigating the viral dynamics in various fields of mathematical biology. In 2009, Pang et.al [4] introduces and discusses the following virus model:

$$\begin{cases} \frac{dS}{dt} = r - dS - mSV, \\ \frac{dI}{dt} = mSV - \xi I, \\ \frac{dV}{dt} = -nV + k\xi I \end{cases}$$
(1)

where S(t) denotes the populations of the uninfected cells, I(t) denotes the infected cells that produce virus, V(t) denotes the free virus particles and r, d, m, n, ξ, k are positive constants. In the model, susceptible host cells are generated at a constant rate r, die at a rate dS and become infected by