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COMPACTIFICATION AND DIVERGENCE OF SOLUTIONS OF POLYNOMIAL FINITE DIFFERENCE SYSTEMS OF EQUATIONS

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Abstract. Polynomial systems of difference systems of equations are studied via an unconventional compactification . It is shown that a polynomial system always possesses at least one fixed point. The fixed point being either finite or infinite, namely ∞p. Linearization of solutions about ∞p , leads to a certain explicit formula for the Jacobian A. A naive expectation that the Jacobian A, solely depends on the highest degree term of the nonlinear system of the difference equations is shown to be false. It is proven that a polynomial difference system that possesses a generic fixed point at infinity, always possesses solution sequences that blows up. An estimate of the rate of blow up is obtained. Then, no finite fixed point is globally asymptotically stable. Applications are provided. They point out to certain nonlinear instabilities of explicit Runga-Kutta methods. Comparisons with analogous results in the realm of polynomial differential equations are also discussed.

Keywords. Nonlinear; Difference systems; Polynomial; Multi dimensional; Numerical Ordinary Differential Equations; Runga-Kutta methods; Chaotic; Compactification; Compacted Equation; Divergence; Fixed point at infinity; Asymptotic Stability; Global; Global asymptotically stability; Jacobian.

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1 Introduction and comparisons with dynamical systems

The purpose of this study is to extend and implement the notions of compactification applied by [14] in the setting of autonomous polynomial differential systems, to the setting of polynomial difference systems of equations. It goes without saying that a global study of autonomous systems of difference equations would be incomplete without the study of their fixed points at infinity. It is noteworthy that in the realm of ordinary differential equations there are quite a few textbooks and works that treat critical points at infinity via compactification. See e.g. [3, 4, 28, 29, 32, 31]. However, I could not find an analogous study of compactification in the majority of text books and monographs that I am familiar with, in the realm of finite difference systems