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AN INTEGRABLE HIERARCHY AND EXPANDING INTEGRABLE SYSTEMS AS WELL AS HAMILTONIAN STRUCTURE

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Abstract. A Lie algebra $sl(3)$ is first presented for which an isospectral Lax pair is introduced. The compatibility condition of the Lax pair gives rise to a new integrable hierarchy with three potential functions, call it a heat hierarchy. As its reduced cases, we present two integrable systems. The first is a generalized heat conduction equation, the second is a generalized AKNS-type hierarchy whose three kinds of Darboux transformations are obtained, which are powerful tools for generating soliton solutions of the nonlinear evolution equations from the generalized AKNS-type hierarchy. In addition, we derive the Hamiltonian structure of the heat hierarchy. Finally, by employing an enlarged Lie algebra of the Lie algebra $sl(3)$, two isospectral problems are introduced whose compatibility condition leads to an integrable hierarchy which is an integrable couplings of the heat hierarchy. The Hamiltonian structure of the integrable couplings is worked out by taking use of the variational identity.

Keywords. Lie algebra, loop algebra, integrable system, Darboux transformation, AKNS hierarchy

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1 Introduction

Research for new integrable systems has been an important aspect of soliton theory. Many methods for generating integrable systems were presented in[1,2]. Ablowitz, et al[3] developed a formalism for deriving a four dimensional integrable hierarchy of commuting nonlinear flows containing the self-dual Yang-Mills flow. Furthermore, Ablowitz, et al[4] took use of the self-dual Yang-Mills equations by reduced ways to obtain some important equations with physical backgrounds, such as the KdV equation, Sine-Gordan equation, and so on. Chakravarty, Kent and Newman[5] utilized the self-dual Yang-Mills equations to obtain some (2+1)-dimensional integrable systems, such as the forced Burgers equation, KP equation and MKP equation. A P Fordy[6]