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LIMITING RELAXED CONTROLS AND AVERAGING OF SINGULARLY PERTURBED DETERMINISTIC CONTROL SYSTEMS

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Abstract. We consider a deterministic controlled dynamics in which the state variables evolve on two different time scales: the velocity of the *fast* variables is proportional to a positive parameter ϵ^{-1}. By using results in homogenization of Hamilton-Jacobi equations we show that, under suitable controllability assumptions, the behavior of the dynamics as $\epsilon \to 0$ is governed by a limit optimal control problem, just for the *slow* variables, obtained by averaging the original vector field with certain probability measures, called limiting relaxed control. As an application, we study the homogenization of fronts undergoing fast oscillations.

Keywords. Singular perturbation; Control systems; Viscosity solutions; Hamilton-Jacobi equations; Limiting relaxed controls; Occupational measures; Differential inclusions; Moving interfaces.

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Introduction

In this paper we consider the following deterministic singularly perturbed control system:

$$\begin{cases} \dot{x}_t = f(x_t, y_t, a_t) & x_0 = x \\ \epsilon\dot{y}_t = g(x_t, y_t, a_t) & y_0 = y \end{cases} \tag{S_ϵ}$$

where $x \in \mathbb{R}^N$, $y \in \mathbb{R}^M$. The functions a_t are *controls*, namely measurable functions defined for any $t > 0$ and valued in a given compact metric space A. We denote by $\mathcal{A}$ the set of such functions. The notation (x_t, y_t) refers to the state of the solution of (S_ϵ) at time t. The following assumptions on the data are standard (see [1], [2], [9], [17]) and are supposed to hold throughout the paper without any further mention:

1. the functions f, and g are bounded and uniformly continuous in $\mathbb{R}^N \times \mathbb{R}^M \times A$, with values, respectively in $\mathbb{R}^N$, and $\mathbb{R}^M$;