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THE ASYMPTOTICS OF GLOBAL SOLUTIONS FOR SEMILINEAR WAVE EQUATIONS IN TWO SPACE DIMENSIONS

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Abstract. The objective of this paper is to establish an asymptotic theory of global solutions for a class of semilinear wave equations in two space dimensions. The validity of formal approximations for time $t \to \infty$ is discussed in the classical sense of C^2, and it is found that the global solution decays like $(1+t+|x|)^{-k}(0 < k < \frac{1}{2})$..

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1 Introduction

In recent years, many attempts have been made to study semilinear wave equations subject to small initial data. A typical problem of this type is as follows

$$u_{tt} - \Delta u = F(u), \quad t > 0, \quad x \in R^2, \tag{1}$$
$$u(0,x) = \varepsilon u_0(x,\varepsilon), \quad u_t(0,x) = \varepsilon u_1(x,\varepsilon), \quad x \in R^2, \tag{2}$$

where u is a real-valued function, $\Delta = \sum_{i=1}^{2} \frac{\partial^2}{\partial x_i^2}$, ε is a parameter with $0 < |\varepsilon| << 1$, $F(u)$, $u_0(x,\varepsilon)$ and $u_1(x,\varepsilon)$ satisfy certain assumptions. The global solvability of the problem defined by (1)–(2) for a compactly supported initial data has been studied by John [6] and Glassey [7] for the case where the nonlinear term $F(u) = |u|^p$, while Glassey has proved the existence of the global classical solution for the above problem when $p > \frac{3+\sqrt{17}}{2}$ and have showed that the solution blows up in a finite time if $1 < p < \frac{3+\sqrt{17}}{2}$. Some similar results for non-compactly supported initial data and more general nonlinear terms have also been established by Kubota [2], John [5].

In this paper, we consider a different type of initial value problems associated with semilinear wave equations, namely, we allow the initial data