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NONLINEAR VARIATION OF PARAMETERS AND AN APPLICATION FOR INTEGRODIFFERENTIAL EQUATIONS WITH INITIAL TIME DIFFERENCE

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Abstract. The relationship between an unperturbed nonlinear integrodifferential system and a perturbed nonlinear integrodifferential system have been investigated that have different initial positions and an initial time difference (ITD). Variation of parameter techniques are employed to obtain analytical formulae and to define Lyapunov-like functions. A simple application of nonlinear variation of parameters formula is given to indicate its usefulness.

Keywords. initial time difference, nonlinear integrodifferential systems, variation of parameters, Lyapunov-like functions, exponentially asymptotically stability.
AMS (MOS) subject classification: 34D10, 34D99.

1 Introduction

The method of variation of parameters formulae [1-7] (VPF) has been very useful in the qualitative theory of differential equations [1, 2, 4, 5] and nonlinear integrodifferential equations [2, 3, 6] since it is a practical tool in the investigation of the properties of solutions. Recently in [4, 5, 7], the study of initial value problems with an ITD has been initiated and the corresponding theory of differential inequalities has been investigated. Below, we shall derive VPF showing the relationship between: nonlinear unperturbed integrodifferentia systems with different initial conditions and nonlinear unperturbed and perturbed integrdifferential systems with different initial conditions. Further, we state generalized VPF relating solutions of a construction of Lyapunov-like functions. Finally, we give a result for the exponentially asymptotically stability as an application of nonlinear variation of parameter formulae to approach the qualitative behavior of perturbed integrodifferential equation with respect to unperturbed integrodifferential equation with the different initial time and position.

2 Notation and Definitions

Let us consider the nonlinear integrodifferential equations

$$Dx(t) = f(t, x(t)) + \int_{t_0}^{t} g(t, s, x(s))ds, x(t_0) = x_0 \tag{2.1}$$

for $t \geq t_0$, $t_0 \in \mathbb{R}_+$ and