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A NEW APPROACH TO STUDY LIMIT CYCLES ON A CYLINDER

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Abstract. We present a new approach to study limit cycles of planar systems of autonomous differential equations with a cylindrical phase space Z. It is based on an extension of the Dulac function which we call Dulac-Cherkas function Ψ. The level set $W := \{\varphi, y) \in Z : \Psi(\varphi, y) = 0\}$ plays a key role in this approach, its topological structure influences existence, location and number of limit cycles. We present two procedures to construct Dulac-Cherkas functions. For the general case we describe a numerical approach based on the reduction to a linear programming problem and which is implemented by means of the computer algebra system Mathematica. For the class of generalized Liénard systems we present an analytical approach associated with solving linear differential equations and algebraic equations.

Keywords. Location and number of limit cycles of first and second kind, Dulac-Cherkas function.

AMS (MOS) subject classification: 34C05 34C07.

1. Introduction

We consider systems of two scalar autonomous differential equations

$$\frac{dx}{dt} = P(x, y), \qquad \frac{dy}{dt} = Q(x, y), \tag{1.1}$$

where P and Q are periodic in x with period 2π. Under this assumption we can identify the phase space of (1.1) with the cylinder $Z := S^1 \times R$, where S^1 is the unit circle. Interpreting x as arclength φ on S^1 we will use for the sequel the notation

$$\frac{d\varphi}{dt} = P(\varphi, y), \qquad \frac{dy}{dt} = Q(\varphi, y). \tag{1.2}$$

Let γ_1 and γ_2 be two closed curves on Z which do not intersect and which are not contractible to a point, that is, they surround the cylinder Z. We denote by Ω the finite region on Z bounded by γ_1 and γ_2 .

An isolated periodic solution of (1.2) with some minimal period is called a