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CHARACTERIZATION OF A LOCAL QUADRATIC GROWTH OF THE HAMILTONIAN FOR CONTROL CONSTRAINED OPTIMAL CONTROL PROBLEMS

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Abstract. We consider an optimal control problem with inequality control constraints given by smooth functions satisfying the hypothesis of linear independence of gradients of active constraints. For this problem, we formulate a generalization of strengthened Legendre condition and prove that this generalization is equivalent to the condition of a local quadratic growth of the Hamiltonian subject to control constraints.

Keywords. Pontryagin's principle, Legendre condition, Hamiltonian, control constraints, quadratic growth, sufficient optimality conditions

AMS (MOS) subject classification: 49K15, 90C46.

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