Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **19 (2012)** 225-265
Copyright ©2012 Watam Press http://www.watsci.org

A GENERIC TURNPIKE RESULT FOR A CLASS OF DISCRETE-TIME OPTIMAL CONTROL SYSTEMS

Alexander J. Zaslavski[1]

[1]Department of Mathematics
The Technion-Israel Institute of Technology, 32000 Haifa, Israel

Email:ajzasl@tx.technion.ac.il

Abstract. In the present paper we establish a turnpike property of approximate solutions for a general class of discrete-time control systems without discounting and with a compact metric space of states. This class of control systems is identified with a complete metric space of objective functions. We show that for a generic objective function approximate solutions of the corresponding control system possess the turnpike property.

Keywords. Good function, optimal solution, trajectory, turnpike property.

AMS (MOS) subject classification: 49J99.

References

[1] S. Aubry and P. Y. Le Daeron, The discrete Frenkel-Kontorova model and its extensions I, *Physica D*, **8**, (1983) 381-422.

[2] J. Baumeister, A. Leitao and G. N. Silva, On the value function for nonautonomous optimal control problem with infinite horizon, *Systems Control Lett.*, **56**, (2007) 188-196.

[3] J. Blot, Infinite-horizon Pontryagin principles without invertibility, *J. Nonlinear Convex Anal.*, **10**, (2009) 177-189.

[4] J. Blot and P. Cartigny, Optimality in infinite-horizon variational problems under sign conditions, *J. Optim. Theory Appl.*, **106**, (2000) 411-419.

[5] P. Cartigny and A. Rapaport, Nonturnpike optimal solutions and their approximations in infinite horizon, *J. Optim. Theory Appl.*, **134**, (2007) 1-14.

[6] D. Gale, On optimal development in a multi-sector economy, *Review of Economic Studies*, **34**, (1967) 1-18.

[7] A. Leizarowitz, Infinite horizon autonomous systems with unbounded cost, *Appl. Math. and Opt.*, **13**, (1985) 19-43.

[8] A. Leizarowitz, Tracking nonperiodic trajectories with the overtaking criterion, *Appl. Math. and Opt.*, **14**, (1986) 155-171.

[9] A. Leizarowitz and V. J. Mizel, One dimensional infinite horizon variational problems arising in continuum mechanics, *Arch. Rational Mech. Anal.*, **106**, (1989) 161-194.

[10] V. Lykina, S. Pickenhain and M. Wagner, Different interpretations of the improper integral objective in an infinite horizon control problem, *J. Math. Anal. Appl*, **340**, (2008) 498-510.

[11] V. L. Makarov and A. M. Rubinov, Mathematical Theory of Economic Dynamics and Equilibria, Springer-Verlag, 1977.

[12] A. B. Malinowska, N. Martins and D. F. M. Torres, Transversality conditions for infinite horizon variational problems on time scales, *Optim. Lett.*, **5**, (2011) 41-53.

[13] L. W. McKenzie, Turnpike theory, *Econometrica*, **44**, (1976) 841-866.

[14] B. S. Mordukhovich, Minimax design for a class of distributed parameter systems, *Automat. Remote Control*, **50**, (1990) 1333-1340.

[15] B. S. Mordukhovich and I. Shvartsman, Optimization and feedback control of constrained parabolic systems under uncertain perturbations, *Optimal Control, Stabilization and Nonsmooth Analysis, Lecture Notes Control Inform. Sci.*, Springer, 2004, 121-132.

[16] J. Moser, Minimal solutions of variational problems on a torus, *Ann. Inst. H. Poincare, Analyse Nonlineare*, **3**, (1986) 229-272.

[17] E. Ocana Anaya, P. Cartigny and P. Loisel, Singular infinite horizon calculus of variations. Applications to fisheries management, *J. Nonlinear Convex Anal.*, **10**, (2009) 157-176.

[18] S. Pickenhain, V. Lykina and M. Wagner, On the lower semicontinuity of functionals involving Lebesgue or improper Riemann integrals in infinite horizon optimal control problems, *Control Cybernet.*, **37**, (2008) 451-468.

[19] A.M. Rubinov, Economic dynamics, *J. Soviet Math.*, **26**, (1984) 1975-2012.

[20] P. A. Samuelson, A catenary turnpike theorem involving consumption and the golden rule, *American Economic Review*, **55**, (1965) 486-496.

[21] A. J. Zaslavski, Ground states in Frenkel-Kontorova model, *Math. USSR Izvestiya*, **29**, (1987) 323-354.

[22] A. J. Zaslavski, Optimal programs on infinite horizon 1, *SIAM Journal on Control and Optimization*, **33**, (1995) 1643-1660.

[23] A. J. Zaslavski, Optimal programs on infinite horizon 2, *SIAM Journal on Control and Optimization*, **33**, (1995) 1661-1686.

[24] A. J. Zaslavski, Turnpike Properties in the Calculus of Variations and Optimal Control, Springer, New York, 2006.

[25] A. J. Zaslavski, Turnpike results for a discrete-time optimal control system arising in economic dynamics, *Nonlinear Analysis*, **67**, (2007) 2024-2049.

[26] A. J. Zaslavski, Two turnpike results for a discrete-time optimal control system, *Nonlinear Analysis*, **71**, (2009) 902-909.

[27] A. J .Zaslavski, Stability of a turnpike phenomenon for a discrete-time optimal control system, *J. Optim. Theory Appl.*, **145**, (2010) 597-612.

[28] A. J. Zaslavski, Overtaking optimal solutions for a class of infinite horizon discrete-time optimal control problems. *Dynamics of Continuous, Discrete and Impulsive Systems, Ser. B: Appl. Algorithms*, **17**, (2010), 607-620.

[29] A. J. Zaslavski, Turnpike properties of solutions for a class of optimal control problems with applications to a forest management problem. *Dynamics of Continuous, Discrete and Impulsive Systems, Ser. B: Appl. Algorithms*, **18**, (2011), 435-459.

Received January 2011; revised March 2011; revised August 2011.

email: journal@monotone.uwaterloo.ca
http://monotone.uwaterloo.ca/∼journal/