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RELAXATION OF OPTIMAL CONTROL PROBLEMS AND LINEAR-QUADRATIC SYSTEMS

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Abstract. The paper suggests an approach to characterizing global solutions for optimal control problems with integral objective functions. The approach is based on relaxation of the system's states to probability measures on the system's state space. The associated relaxed control problem falls, typically, to the scope of convex optimization problems with linear equality constraints. Under additional conditions assuming, in particular, that the objective function and state equation are linear-quadratic in the state variable, the equivalency of the original and relaxed problems is proved and a successive solution approximation method is constructed.

Keywords. global optimization, non-convex optimization, optimal control, relaxation of optimization problems, successive optimization methods

References

[1] D.P. Bertsecas, Constrained Optimization and Lagrange Multipliers Methods, Academic Press, New York, 1982.

[2] D.A. Carlson and G. Leitmann, An Equivalent problem approach to absolute extrema for calculus of variations problems with differential constraints, *Dynamics of Continuous, Discrete and Impulsive Systems, Series B*, **18** (2011), 1-15.

[3] L. Cesari, *Optimization ?Theory and Applications. Problems with Ordinary Differential Equations*, Springer, New York, 1983.

[4] R. Horst, *Introduction to Global Optimization (Nonconvex Optimization and Its Applications)*, Springer, New York, 2002.

[5] N.N.Krasovskiy and A.I.Subbotin, *Positional Differential Games*, Na- uka, Moscow, 1974.

[6] A.V.Kryazhimskiy, Convex optimization via feedbacks, *SIAM J. Control Optimiz.*, **37** (1999), 278-302.

[7] A.V.Kryazhimskiy, Optimization problems with convex epigraphs. Application to optimal control, *Intern. J. Appl. Math. Comp. Sci.*, **11** (2001), 101-129.

[8] A.V.Kryazhimskiy and A. Ruszczynski, Constraint aggregation in in- finite-dimensional spaces and applications, *Math. Operat. Research*, **26** (2001), 769-795.

[9] A.V.Kryazhimskiy and Yu.S.Osipov, To a regularization of a convex extremal problem with inaccurately given constraints. An application to an optimal control problem with state constraints. In: *Some Methods of Positional and Program Control*, Urals. Sci. Center, Sverdlovsk, 1987.

[10] A.V.Kryazhimskiy and Yu.S.Osipov, Extremal problems with separable graphs, *Cybernetics Syst. Anal.*, 2002, No 2, 32-55.

[11] A.V.Kryazhimskiy and V.I. Maksimov, A solution algorithm for problems of optimal control in Hilbert spaces, *J. Mathem. Sci.*, **121** (2004), 2226-2247.

[12] A.V.Kryazhimskiy and Yu.S.Osipov, Method of extremal shift and optimization problems, *Proc. Inst. Math. Mechan. Urals Branch, Russian Acad. Sci.*, **10** (2004), 83-105.

[13] A.S.Matveyev and V.A.Yakubovich, Nonconvex global optimization problems, *Algebra and Analysis*, **4:6** (1992), 189-219.

[14] B.S.Mordukhovich, *Variational Analysis and Generalized Differentiation*, Springer, Berlin, 2006.

[15] P.M.Pardalos and H.E.Romeijn (eds), *Handbook of Global Optimization. Vol 2. Nonconvex Optimization and Its Applications*, Kluwer Academic Publishers, Dordrecht, 2002.

[16] L.S.Pontryagin, V.G.Boltyanskij, R.V.Gamkrelidze and E.F.Mishche- nko, *The Mathematical Theory of Optimal Processes*, Pergamon, Oxford, 1964.

[17] R.T.Rockafellar and J-B.Wets, *Variational Analysis*, Springer, Berlin, 1998.

[18] A.N.Tikhonov and V.Ya.Arsenin, *Solution Methods for Ill-posed Problems*, Nauka, Moscow, 1979.

[19] J.Warga, *Optimal Control of Differential and Functional Equations.* Academic Press, New York, 1972.

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