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APPROXIMATE SOLUTION OF THE HJB INEQUALITY RELATED TO THE INFINITE HORIZON OPTIMAL CONTROL PROBLEM WITH DISCOUNTING[1]

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Abstract. This paper is focusing on finding smooth approximate solutions of the HJB inequality that corresponds to the infinite horizon optimal control problem with discounting. We establish that such approximate solutions exist (under a simple controllability type condition) and that they can be used for construction of near optimal controls. We also show that these approximate solutions of the HJB inequality can be found by solving certain semi-infinite linear programming problems and we propose an algorithm for the solution of the latter. We discuss a numerical solution of a non-trivial optimal control problem obtained with the help of a software implementation of the new algorithm.

Keywords. Linear programming approach to optimal control problems; numerical solution of optimal control problems; HJB inequality; duality; semi-infinite linear programming; discounted occupational measures.

AMS (MOS) subject classification: 34E15, 34C29, 34A60, 93C70

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