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SYNTHESIS OF OPTIMAL CONTROLLED TRAJECTORIES WITH CHATTERING ARCS

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Abstract. A field theoretic approach that is based on the method of characteristics is outlined to prove the optimality of a synthesis of controlled trajectories that contains chattering arcs. Since the corresponding controls are not piecewise continuous - they switch infinitely many times on finite intervals - the classical argument by Boltyansky to prove differentiability of the associated value function is no longer applicable. This difficulty is overcome by matching the *two* parameterized families of extremal controlled trajectories that correspond to the constant controls $u = \pm 1$ at the switching surfaces in a continuously differentiable way.

Keywords. optimal control, Fuller problem, chattering controls, synthesis, tumor anti-angiogenesis.

AMS (MOS) subject classification: 49N35, 49L20, 93C10

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