# SOME RETROSPECTIVE REMARKS ON PATHWISE ASYMPTOTIC OPTIMALITY

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**Abstract.** This note contains a brief survey and some remarks regarding the notion of pathwise asymptotic optimality in infinite horizon stochastic optimization problems.

**Keywords.** Controlled processes, pathwise optimality, asymptotic optimality, optimality almost surely, optimality in probability, optimality in distribution.

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