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A MORSE THEORETIC APPROACH TO THE EXISTENCE OF MULTIPLE SOLUTIONS FOR NONLINEAR PERIODIC PROBLEMS

Sophia Th. Kyritsi[1], Donal O'Regan[2], and Nikolaos S. Papageorgiou[3]

[1]Department of Mathematics
Hellenic Naval Academy, Piraeus 18539, Greece

[2]Department of Mathematics
National University of Ireland, University Road, Galway, Ireland

[3]Department of Mathematics
National Technical University, Athens 15780, Greece
Corresponding author email:npapg@math.ntua.gr

Abstract. We consider a nonlinear periodic problem driven by the scalar p-Laplacian and with a Carathéodory nonlinearity which asymptotically at infinity exhibits a p-linear growth. Using minimax arguments, together with truncation techniques and methods from Morse theory, we show that the problem has three nontrivial solutions. In the semilinear case (i.e., $p=2$), we show using Morse theory, that the problem has four nontrivial solutions.

Keywords. Periodic scalar p-Laplacian, mountain pass theorem, Morse theory, critical groups, multiple solutions.

AMS (MOS) subject classification: 34B15, 34B18, 34C25, 58E05.

1 Introduction

In this paper, we study the following nonlinear periodic problem driven by the scalar p-Laplacian:

$$\left\{ \begin{array}{c} -\big(|x'(t)|^{p-2}x'(t)\big)' = f\big(t,x(t)\big) \quad \text{a.e. on } T=[0,b], \\ x(0)=x(b),\ x'(0)=x'(b),\ 1<p<\infty. \end{array} \right\} \tag{1}$$

Here $f(t,x)$ is a Carathéodory nonlinearity, which near $\pm\infty$ exhibits a $(p-1)$-linear growth. Our aim is to prove a multiplicity theorem, establishing the existence of three nontrivial solutions for problem (1).

Multiplicity results for periodic problems with the scalar p-Laplacian, were proved by Aizicovici–Papageorgiou–Staicu [2], del Pino–Manásevich–Murúa [10], Gasiński–Papageorgiou [14], Papageorgiou–Papageorgiou [22] and Yang [25].

In del Pino–Manásevich–Murúa [10], the authors consider a continuous nonlinearity $(t,x) \longrightarrow f(t,x)$ and in their approach, they combine degree theoretic arguments based on the Leray–Schauder degree map, with the parallel use of time maps. They prove existence and multiplicity results establishing the existence of