THREE NESTED LIMIT CYCLES IN DISCONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEMS WITH TWO ZONES

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Abstract. In this paper we study a planar piecewise linear differential system formed by two regions separated by a straight line so that one system has a real unstable focus and the other a virtual stable focus which coincides with the real one. This system was introduced in a very recent paper (On the number of limit cycles in general planar piecewise linear systems, *Discrete and Continuous Dynamical Systems-A* **32**, 2012, pp. 2147–2164) by S.-M. Huan and X.-S. Yang, who numerically showed that it can exhibit 3 limit cycles surrounding the real focus. This is the first example that a non-smooth piecewise linear differential system with two zones can have 3 nested limit cycles of crossing type surrounding a unique equilibrium. We provide a rigorous computer assisted proof of the quoted numerical result.

Keywords. non–smooth differential system, limit cycle, piecewise linear differential system.

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References

[1] A. Andronov, A. Vitt and S. Khaikin, Theory of Oscillations, Pergamon Press, Oxford, 1966.

[2] J. Bernat and J. Llibre, Counterexample to Kalman and Markus–Yamabe conjectures in dimension larger than 3, *Dynamics of Continuous, Discrete and Impulsive Systems* **2**, (1996) 337–379.

[3] S. Coombes, Neuronal networks with gap junctions: A study of piecewise linear planar neuron models, SIAM Applied Mathematics **7**, (2008) 1101–1129.

[4] M. di Bernardo, C.J. Budd, A. R. Champneys, P. Kowalczyk, Piecewise-Smooth Dynamical Systems: Theory and Applications, Appl. Math. Sci. Series 163, Springer–Verlag, London, 2008.

[5] Z. Du, Y. Li, W. Zhang, Bifurcation of periodic orbits in a class of planar Filippov system, *Nonlinear Anal.* **69**, (2008) 3610–3628.

[6] A. F. Filippov, Differential Equations with Discontinuous Right-Hand Sides, Kluwer Academic, Dordrecht, 1988.

[7] E. Freire, E. Ponce, F. Rodrigo and F. Torres, Bifurcation sets of continuous piecewise linear systems with two zones, *Int. J. Bifurcation and Chaos* **8**, (1998) 2073–2097.

[8] E. Freire, E. Ponce, F. Rodrigo and F. Torres, Bifurcation sets of symmetrical continuous piecewise linear systems with three zones, *Int. J. Bifurcation and Chaos* **12**, (2002) 1675–1702.

[9] E. Freire, E. Ponce and F. Torres, Canonical Discontinuous Planar Piecewise Linear Systems, *SIAM J. Applied Dynamical Systems* **11**, (2012) 181–211.

[10] F. Giannakopoulos and K. Pliete, Planar Systems of Piecewise Linear Differential Equations with a Line of Discontinuity, *Nonlinearity* **14**, (2001) 1611–1632.

[11] M. Han and W. Zhang, On Hopf bifurcation in non–smooth planar systems, *J. of Differential Equations* **248**, (2010) 2399–2416.

[12] S.-M. Huan and X.-S. Yang, On the number of limit cycles in general planar piecewise linear systems, *Discrete and Continuous Dynamical Systems-A* **32**, (2012) 2147–2164.

[13] E. Isaacson and H.B. Keller, Analysis of numerical methods, John Wiley and Sons, New York, 1966.

[14] J. Stoer and R. Burlisch, Introduction to Numerical Analysis, Springer–Verlag, New York, 1980.

[15] A. Tonnelier, The McKean's caricature of the FitzHugh-Nagumo model I. The space-clamped system, *SIAM J. Appl. Math.* **63**, (2003) 459–484.

[16] A. Tonnelier and W. Gerstner, Piecewise linear differential equations and integrate-and-fire neurons: Insights from two-dimensional membrane models, *Phys Rev. E* **67**, (2003) 021908.