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THE GLOBAL SOLUTIONS FOR A SHALLOW WATER EQUATION WITHOUT PEAKONS

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Abstract. A nonlinear shallow water equation including the Camassa-Holm and Degasperis-Procesi equations without peakon solutions is investigated. The local well-posedness of solutions for the equation in Sobolev space $H^s(R)$ with $s > \frac{3}{2}$ is established. The existence and uniqueness of its global solutions are shown to be true in space

$$C([0,\infty); H^s(R)) \bigcap C^1([0,\infty); H^{s-1}(R))$$

under certain assumptions.

Keywords. Global existence; Shallow water model.

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References

[1] B. Fuchssteiner and A.S. Fokas, Symplectic structures, their Backlund transformation and hereditary symmetries, *Phys. D*, **4**, (1981) 821-831.

[2] R. Camassa and D. Holm, An integrable shallow water equation with peaked solitons, *Phys. Rev. Lett.*, **71**, (1993) 1661-1664.

[3] A. Constantin and D. Lannes, The hydro-dynamical relevance of the Camassa-Holm and Degasperis-Procesi equations, *Arch. Ration. Mech. Anal.*, **193**, (2009) 165-186.

[4] R.S. Johnson, Camassa-Holm, Korteweg de Vries and related models for water waves, *J. Fluid Mech.*, **455**, (2002) 63-82.

[5] A. Constantin, On the scattering problem for the Camassa-Holm equation, *Proc. R. Soc. Lond.*, **457**, (1990) 953-970.

[6] H.P. McKean, Fredholm determinants and the Camassa-Holm hierarchy, *Comm. Pure Appl. Math.*, **56**, (2003) 638-680.

[7] A. Constantin and J. Escher, Global existence and blow-up for a shallow water equation, *Ann. Scuola Norm. Sup. Pisa Cl. Sci.*, **26**, (1998) 303-328.

[8] A. Constantin and B. Kolev, Geodesic flow on the diffeomorphism group of the circle, *Comment. Math. Helv.* , **58**, (2003) 787-804.

[9] A. Constantin, T. Kappeler, B. Kolev and P. Topalov, On geodesic exponential maps of the Virasoro group, *Ann. Global. Anal. Geom.*, **31**, (2007) 155-180.

[10] O. Glass, Controllability and asymptotic stabilization of the Camassa-Holm equation, *J. Differential Equations*, **245**, (2008) 1584-1615.

[11] J. Lenells, Traveling wave solutions of the Camassa-Holm equation, *J. Differential Equations*, **217**, (2005) 393-430.

[12] A. Degasperis, D. Holm, A. Hone, A new integral equation with peakon solutions, *Theor. Math. Phys.*, **133**, (2002) 1461-1472.

[13] H.R. Dullin, G.A. Gottwald and D.D. Holm, Camassa-Holm, Korteweg-de Vries-5 and other asymptotically equivalent equations for shallow water waves, *Fluid Dynam. Research*, **33**, (2003) 73-79.

[14] V.O. Vakhnenko and E.J. Parkes, Periodic and solitary-wave solutions of the Degasperis-Procesi equation, *Chaos Solitons Fractals*,**20**, (2004) 1059-1073.

[15] D.D. Holm and M.F. Staley, Wave structure and nonlinear balances in a family of evolutionary PDEs, *SIAM J. Appl. Dyn. Syst.*, **2**, (2003) 323-380.

[16] H. Lundmark, J. Szmigielski, Multi-peakon solutions of the Degasperis-Procesi equation, *Inverse Problems*, **19**, (2003) 1241-1245.

[17] Z.W. Lin and Y. Liu, Stability of peakons for the Degasperis-Procesi equation, *Comm. Pure. Appl. Math.*, **62**, (2009) 125-146.

[18] Z.Y. Yin, On the Cauchy problem for an intergrable equation with peakon solutions, *Illinois J. of Math.*, **47**, (2003) 649-666.

[19] Z.Y. Yin, Global weak solutions for a new periodic integrable equation with peakon solutions, *J. Funct. Anal.*, **212**, (2004) 182-194.

[20] J. Lenells, Traveling wave solutions of the Degasperis-Procesi equation, *J. Math. Anal. Appl.*, **306**, (2005) 72-82.

[21] Y. Matsuno, Multisoliton solutions of the Degasperis-Procesi equation and their peakon limit, *Inverse Problems*, **21**, (2005) 1553-1570.

[22] O.G. Mustafa, A note on the Degasperis-Procesi equation, *J. Nonlinear Math. Phys.*, **12**, (2005) 10-14.

[23] G.M. Coclite and K.H. Karlsen, On the well-posedness of the Degasperis-Procesi equation, *J. Funct. Anal.*, **223**, (2006) 60-91.

[24] S.Y. Lai and Y.H. Wu, Global solutions and blow-up phenomena to a shallow water equation, *J. Differential Equations*, **249**, (2010) 693-706.

[25] S.Y. Lai and Y.H. Wu, A model containing both the Camassa-Holm and Degasperis-Processi equations, *J. Math. Anal. Appl.*, **371**, (2011) 458-469.

[26] C.B. Ruscitti, Existence and uniqueness for viscous Boussinesq system in abstract Banach spaces, *Dynamics of Continous, Discrete and Impulsive systems*, **17**, (2010) 659-675

[27] I. Altun and D. Turkoglu, An existence theorem for the common solutions for a pair of integral inclusions, *Dynamics of Continous, Discrete and Impulsive systems*, **18**, (2011) 135-147

[28] T. Kato, Quasi-linear equations of evolution with applications to partial differential equations, Spectral Theory and Differential Equations, *Lecture Notes in Math. Springer Verlag, Berlin*, **448**,(1975) 25-70.

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