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A FAMILY OF NOVEL CHAOTIC AND HYPERCHAOTIC ATTRACTORS FROM DELAY DIFFERENTIAL EQUATION

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Abstract. In this paper, a family of novel chaotic and hyperchaotic attractors are constructed utilizing a first-order delay differential equation (DDE). Dynamical analysis exhibits that Hopf bifurcation occurs at the non-trivial equilibrium points of the system when the time delay is properly selected. Bifurcation diagram and Lyapunov spectra further verify that the system behaves alternately in chaotic and periodic manners with the system parameter varying. By controlling the system parameter to increase the number of equilibrium points, a family of complex chaotic and hyper-chaotic attractors arise. Furthermore, we present a more general form of DDE and simulate its various chaotic dynamics under different-sign system parameters. The boundedness of this general DDE is studied in detail and finally, a possible circuit implement for these new attractors is proposed.

Keywords. Chaotic attractor, delay differential equation, Lyapunov exponent, Hopf bifurcation, chaos circuit.

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