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New Forced Oscillation Criteria for Second Order Half-Linear Differential Equations[1]

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Abstract. Using generalized variational principle and Riccati technique, new oscillation criteria for forced second order half-linear differential equation are established, which improve some recent papers in literature.

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1 Introduction

There have been a lot of papers involved oscillation (see [2, 4, 6-9])for the second order half-linear differential equation without or with forcing term

$$\big(r(t)|y'(t)|^{\alpha-1}y'(t)\big)' + q(t)|y(t)|^{\alpha-1}y(t) = 0, t \geq t_0 \tag{1}$$

and

$$\big(r(t)|y'(t)|^{\alpha-1}y'(t)\big)' + q(t)|y(t)|^{\alpha-1}y(t) = e(t), t \geq t_0 \tag{2}$$

since the foundation work of A. Elbert [4]. Here $r, q, e \in C([t_0, \infty), \mathbb{R})$ with $r(t) > 0$ and $\alpha > 0$ is a constant. The half-linear differential equation (1) has the similar properties to linear differential equation, for example, the Sturm's comparison theorem and separation theorem (see Elbert [4], Li and Yeh [7] for details) are still true for equation (1). In particular, the Riccati type equation (related to (1) by the substitution $w = r|y'|^{\alpha-1}y'/|y|^{\alpha-1}y$)

$$w' + q(t) + \alpha r^{-1/\alpha}(t)|w|^{(\alpha+1)/\alpha} = 0 \tag{3}$$

and the $(\alpha+1)-$degree functional

$$I(y) := \int_a^b \left[q(t)|y|^{\alpha+1} - r(t)|y'|^{\alpha+1}\right] dt \tag{4}$$

play the same role as the classical Riccati equation and quadratic functional in the linear oscillation theory (The proof of the relationship between (1), (3) and (4) is based on the recently established Picone's identity, see [6])

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