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OSCILLATION OF SECOND ORDER EMDEN-FOWLER TYPE NEUTRAL DIFFERENCE EQUATIONS

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Abstract. Some new oscillation criteria for the second order Emden-Fowler type neutral difference equation

 $\triangle (a_n \triangle (x_n + p_n x_{n-k})) + q_n x_{\sigma(n)}^{\alpha} = 0, \quad n \ge n_0$

where k is a positive integer, α is a ratio of odd positive integers and $\sigma(n) \leq n$ are established under the condition $\sum_{n=n_0}^{\infty} \frac{1}{a_n} < \infty$. Examples are provided to illustrate the results.

Keywords. Oscillation, Second order Emden-Fowler neutral difference equation.

AMS (MOS) subject classification: 39A10,39A12.

1 Introduction

In this paper we study the oscillatory behavior of the second order Emden-Fowler type neutral difference equation

$$\triangle (a_n \triangle (x_n + p_n \, x_{n-k})) + q_n \, x_{\sigma(n)}^{\alpha} = 0, \tag{1.1}$$

where $n \in \mathbb{N}(n_0) = \{n_0, n_0 + 1, \ldots\}$, n_0 is a nonnegative integer, k is a positive integer, α is a ratio of odd positive integers, $\{a_n\}, \{p_n\}, \{q_n\}$ are real sequences and $\{\sigma_n\}$ is a sequence of integers. We assume that the following conditions hold:

(c₁)
$$\{a_n\}$$
 is a positive real sequence with $\sum_{n=n_0}^{\infty} \frac{1}{a_n} < \infty$;

(c₂) $\{p_n\}$ is a nondecreasing real sequence with $0 \le p_n < 1$;

- (c₃) $\{q_n\}$ is a nonnegative real sequence and not identically zero for many values of n;
- (c₄) $\{\sigma_n\}$ is a sequence of integers such that $\sigma(n) \leq n, \Delta \sigma(n) > 0$ and $\lim_{n \to \infty} \sigma(n) = \infty$.