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ON INPUT-TO-STATE STABILITY OF NONLINEAR STOCHASTIC HYBRID SYSTEMS

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Abstract. This paper is concerned with the input-to-state stability concept of nonlinear stochastic hybrid systems with bounded disturbance input. The main objective of the paper is to develop Lyapunov-like sufficient conditions guaranteeing the stability property in the pth moment. To control the switching among the system modes, we adopt two switching rules, a newly developed initial-state-dependent dwell-time and Markovian switching. It has been shown that the stability property of individual modes is neither necessary nor sufficient to ensure the stability of the switched system. Implications of these results are also stated, and some examples are worked out to justify the effectiveness of the proposed theoretical results.
Keywords. Nonlinear system, switched system, input-to-state stability, Wiener process, initial-state-dependent dwell-time, Markovian switching.

1 Introduction

Input-to-state stability (ISS) introduced in [25, 29] is a useful tool in analyzing the effect of disturbance input on the behaviour of feedback closed-loop system trajectories. The importance of studying ISS is manyfold. It bridges the gap between the input/output stability and the Lyapunov stability, it has many equivalencies or implications to other stability-like concepts, such as integral-ISS, global asymptotic stability (for zero input), and finite gain with respect to supremum norms and finite L^2, and its applications to linear and nonlinear system and control theory, such as coprime factorization, cascade or feedforward systems, small-gain theorems, and singularly perturbed systems. For further characterizations, applications, and directions on the applications of ISS, one may consult [3, 6, 26, 27, 28, 29, 30] and some other references cited therein.

A large class of natural or human-made systems are properly governed by multi-dynamical subsystems (or modes), such as control systems, robots, thermostats in cooling or heating systems, prey-predator systems with different but finite prey sources, and epidemic disease models. The interchange