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ON A GENERALIZED LOTKA-VOLTERRA SYMBIOSIS SYSTEM

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Abstract. Hirsch et al. give several properties of a generalized Lotka-Volterra symbiosis system in [3, 4]. However, we find counter-examples about the non-existence of sources and the uniqueness of a sink. We show them and give sufficient conditions to establish their properties. Also, we present results on the number of equilibria.

Keywords: symbiosis system, non-existence of sources, uniqueness of a sink

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1 Introduction

In various fields of mathematical biology, population models have been proposed and studied extensively. The predator-prey, the competition and the symbiosis (or the cooperation) are the three most fundamental systems.

May [7] proposed the generalized Lotka-Volterra system

$$\begin{cases} x' = M(x,y)\,x \\ y' = N(x,y)\,y \end{cases} \tag{1}$$

where x and y denote the populations of two species, and M and N, which are sufficient smooth functions, represent the growth rates of x and y respectively. He assumed that two species are in predator-prey relation, that is, M and N satisfy

$$\frac{\partial M}{\partial y}(x,y) < 0 \quad \text{and} \quad \frac{\partial N}{\partial x}(x,y) > 0,$$

and discussed the existence of limit cycles. See [6] for other references.

Hirsch et al. [3, 4] consider (1) in case that two species are in competition, that is, M and N are assumed to satisfy

$$\frac{\partial M}{\partial y}(x,y) < 0 \quad \text{and} \quad \frac{\partial N}{\partial x}(x,y) < 0.$$

Under a few additional qualitative assumptions, they conclude that the populations of two competing species always tend to one of a finite number of