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A SPECTRAL COLLOCATION METHOD BASED ON CHEBYSHEV POLYNOMIALS FOR THE GENERALIZED ZAKHAROV EQUATION

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Abstract. In this paper, we use the spectral collocation method based on Chebyshev polynomials for spatial derivatives and fourth order Runge-Kutta (RK) method for time integration to solve the generalized Zakharov equation (GZE). Firstly, theory of application of Chebyshev spectral collocation method on the GZE is presented. This method yields a system of ordinary differential equations (ODEs). Secondly, we use fourth order RK formula for the numerical integration of the system of ODEs. The numerical results obtained by this way have been compared with the exact solution to show the efficiency of the method.

Keywords. Spectral collocation method, generalized Zakharov equation, Chebyshev polynomials.

AMS (MOS) subject classification: 65N35, 35E15.

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