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LOWER AND UPPER BOUNDS OF SOLUTIONS OF FUNCTIONAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we first give a general theorem on the lower bound of functional differential equations $u'(t) = F(t, u_t)$, $u_t \in \mathbf{C}$. Then obtain the estimate of lower and upper bounds of solutions of the Volterra integro-differential equations:

$$\frac{d}{dt}X(t) = A(t)F(X(t)) + \int_0^t B(t,s)G(X(s))ds + H(t), \quad X \in \mathbf{R^n}, \tag{1}$$

and

$$\frac{d}{dt}X(t) = A(t)F(X(t)) + \int_{t-h}^t B(t,s)G(X(s))ds + H(t) \quad X \in \mathbf{R^n}. \tag{2}$$

Keywords. Lyapunov's Second Method, Boundedness, Stability, Inequalities of Solutions, Volterra Integro-differential Equations, Functional Differential Equations

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