

OPTIMAL GUARANTEED COST CONTROL OF STOCHASTIC DISCRETE-TIME SYSTEMS UNDER MARKOVIAN REGIME SWITCHING

S. Sathananthan¹, M.J. Knap¹, and L.H. Keel²

¹Department of Mathematics and Center of Excellence in ISEM
3500 John A. Merritt Blvd., Campus Box 9501
Tennessee State University, Nashville, TN 37209, U.S.A

²Department of Electrical & Computer Engineering and Center of Excellence in ISEM
Campus Box 9501, Tennessee State University, Nashville, TN 37209, U.S.A

Abstract. A problem of robust guaranteed cost control of stochastic discrete-time systems with parametric uncertainties under Markovian switching is considered. The jump Markovian switching is modelled by a discrete-time Markov chain and the noise or stochastic environmental disturbance is modelled by a sequence of identically independently normally distributed random variables. Using linear matrix inequalities (LMI's) approach, the robust quadratic stochastic stability is obtained. The proposed control law for this quadratic stochastic stabilization result depended on the mode of the system. This control law is developed such that the closed-loop system with a cost function has an upper bound under all admissible parameter uncertainties. The upper bound for the cost function is obtained as a minimization problem. Two numerical examples are given to demonstrate the potential of the proposed techniques and obtained results.

Keywords. Stochastic Stability, Markovian Jump Linear Systems, guaranteed cost, quadratic stabilization, discrete-time stochastic systems.

References

- [1] E.K. Boukas and Z.K. Liu *Deterministic and Stochastic Time Delay Systems*. Birkhauser, Boston, 2002.
- [2] E.K. Boukas. *Stochastic Switching Systems, Analysis and Design*. Birkhauser, Boston, 2006.
- [3] E.K. Boukas and Huaping Liu. Delay-Dependent Stabilization of Stochastic Discrete-Time Systems with Time-Varying Time-Delay. *Proceedings of the 2007 American Control Conference*, New York City, July 11-13, 2007.
- [4] Peng Shi, E.K. Boukas and Yan Shi. On Stochastic Stabilization of Discrete-Time Markovian Jump Systems with Delay in State. *Stochastic Analysis and Applications*, 21(4):935-951, 2003.
- [5] Peng Shi, E.K. Boukas, Yan Shi, and R.K. Agarwal. Optimal Guaranteed Cost Control of Uncertain Discrete Time-delay Systems. *Journal of Computational and Applied Mathematics*, 157:435-451, 2003.
- [6] Y.Y. Cao and J. Lam. Robust H_∞ Control of Discrete-time Markovian Jump Linear Systems with Mode-dependent Time-delays. *Journal of the Franklin Institute*, 336(8): 1263-1281, 1999.
- [7] O.L.V. Costa, M.D. Fragoso and R. P. Marques *Discrete-time Markov Jump Linear Systems*. Springer, 2005. *IEEE Transactions on Automatic Control*, 35:665-672, 1990.
- [8] M.H.A. Davis. *Linear Estimation and Control*. John Wiley and Sons, New York, 1977.
- [9] V. Dragan, T. Morozan, and A. Stoica. *Mathematical Methods in Robust Control of Discrete-Time Linear Stochastic Systems*. Springer, 2010.
- [10] V. Dragan and T. Morozan. Exponential Stability in Mean Square for a General Class of Discrete-Time Linear Stochastic Systems. *Stochastic Analysis and applications*, 26: 495-525, 2008.
- [11] V. Dragan and T. Morozan. Exponential Stability for Discrete-Time Linear Equations Defined by Positive Operators. *Integral Equations and Operator Theory*, 54:465-493, 2006.
- [12] V. Dragan and T. Morozan. Mean Square Exponential Stability for Some Stochastic Discrete-Time Linear Systems. *European Journal of Control*, 4(12):373-395, 2006.
- [13] H. Abou-Kandil, G. Freiling, and G. Jank. On the Solution of Discrete-time Markovian Jump Linear Quadratic Control Problems. *Automatica*, 31,No.5:. 765-768, 1995.
- [14] G.S. Ladde and D.D. Siljak. Multiplex Control Systems: Stochastic Stability and Dynamic Reliability. *International Journal of Control*, 35:515-524, 1983.
- [15] G. Byron and G. S. Ladde. Qualitative properties of stochastic iterative processes under random structural perturbations. *Mathematics and Computers in Simulation*, 67:181-200, 2004.
- [16] Y.S. Lee, O. H. Kwon, and W.H. Kwon. Delay-dependent Guaranteed Cost Control for Uncertain State-delayed Systems. *International Journal of Control, Automation and Systems*, 3,No.4:. 524-532, 2005.
- [17] Yuguang Fang and Kenneth A. Loparo. Stochastic Stability of Jump Linear Systems. *IEEE Transactions on Automatic Control Systems*, Vol.47, No.7, :1204-1208, July 2002.
- [18] M.S. Mahmoud. *Robust Control and Filtering for Time-Delay Systems*. Marcel Dekker, 2000.
- [19] M. Mariton. *Jump Linear Systems*. Marcel Dekker, New York, 1990.
- [20] X. Guan, Z. Lin, and G. Duan. Robust Guaranteed Cost Control for Discrete-time Uncertain systems with Delay. *IEE proceedings*, 146,No.6:. 598-602, 1999.

- [21] X. Guan, J. Wu, C. Long, and P. Shi. Resilient Guaranteed Cost Control for Uncertain Discrete Linear Jump Systems. *International Journal of Systems Science*, 34,No.4:. 283–292, 2003.
- [22] S. Sathananthan. Quantitative analysis of jump Markovian nonlinear stochastic hybrid systems: practical stability. *Nonlinear Studies*, 8:407–428, 2001.
- [23] S. Sathananthan, O. Adetona, C. Beane, and L.H. Keel. Feedback stabilization of Markov jump linear systems with time-varying delay. *Stochastic Analysis and Applications*, 26:577–594, 2008.
- [24] S. Sathananthan, O. Adetona, C. Beane, and L.H. Keel. Delay-dependent stability criteria for Markovian switching networks with time-varying delay. *Stochastic Analysis and Applications*, 27:694–712, 2009.
- [25] S. Sathananthan, C. Beane, G. S. Ladde and L.H. Keel. Stabilization of stochastic systems under Markovian switching. *Nonlinear Analysis: Hybrid Systems*, 4:804–817, 2010.
- [26] S. Sathananthan, M.J. Knap and L.H. Keel. Robust control of stochastic systems with noise dependent states and inputs under Markovian Markovian switching. Proceedings of the 2010 American Control Conference, Baltimore, June 30-July 2, 904–909, 2010.
- [27] Y.S. Wang, L. Xie and C.E. De Souza. Robust Control of a Class of Uncertain Systems. *Systems and Control Letters*, 19:139–149, 1992.
- [28] M. Wu, Y. He, J. She, and G. Liu. Delay-dependent Criteria for Robust Stability of Time-Varying Delay Systems. *Automatica*, 40:1435–1439, 2004.
- [29] G.H. Yang, J.L.Wang, and Y.C. Soh. Guaranteed Cost Control for Discrete-time Linear Systems under Controller Gain Perturbations. *Linear Algebra and Its Applications*, 312:. 161–180, 2000.
- [30] D. Yue, J. Fang, and S. Won. Delay-dependent robust stability of stochastic uncertain systems with time delay and Markovian jump parameters. *Circuits Systems Signal Processing*, 22:351–365, 2003.
- [31] D. Yue and Q. Han. Delay-dependent exponential stability of stochastic systems with time-varying delay, nonlinearity, and Markovian switching. *IEEE Transactions on Automatic Control*, 50:217–222, 2005.

Received October 2012; revised March 2013.

email: journal@monotone.uwaterloo.ca

<http://monotone.uwaterloo.ca/~journal/>