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ON A THIRD ORDER RATIONAL DIFFERENCE EQUATION WITH VARIABLE COEFFICIENTS

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Abstract. In this paper we investigate the solutions of the rational difference equation

$$x_{n+1} = \frac{x_{n-1}x_{n-2}}{x_n(a_n + b_n\, x_{n-1}\, x_{n-2})},\ n \in \mathbb{N}_0$$

where $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$ are real two-periodic sequences and the initial values x_{-2}, x_{-1}, x_0 are non-zero real numbers.

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References

[1] C. Cinar, On the positive solutions of the difference equation $x_{n+1} = \frac{x_{n-1}}{1+x_n x_{n-1}}$, *Appl. Math. Comp.*, **150**, (2004), 21-24, .

[2] C. Cinar, R. Karatas and I. Yalçinkaya, On solutions of the difference equation $x_{n+1} = \frac{x_{n-3}}{-1+x_n x_{n-1} x_{n-2} x_{n-3}}$, *Math. Bohem.*, **132**, (2007), 257-261.

[3] , D. Chen, X. Li and Y. Wang, Dynamics for nonlinear difference equation $x_{n+1} = \frac{\alpha x_{n-k}}{\beta+\gamma x_{n-l}^p}$, *Adv. Differ. Equ.*, Article ID 235691, 13 pages, (2009).

[4] E. M. Elabbasy, H. El-Metwally and E. M. Elsayed, On the difference equation $x_{n+1} = ax_n - \frac{bx_n}{cx_n - dx_{n-1}}$, *Adv. Differ. Equ.*, Article ID 82579, 10 pages, (2006).

[5] E. M. Elabbasy and E. M. Elsayed, On the global attractivity of difference equation of higher order, *Carpathian J. Math.*, **24**, (2008), 45-53.

[6] H. El-Metwally and E. M. Elsayed, Qualitative study of solutions of some difference equations, *Abstr. Appl. Anal.*, Article ID 248291, 16 pages, (2012).

[7] E. M. Elsayed, On the global attractivity and the solution of recursive sequence, *Stud. Sci. Math. Hung.*, **47**, (2010), 401-418.

[8] E. M. Elsayed, Expressions of solutions for a class of differential equations, *An. Ştiint,. Univ. "Ovidius" Constanta. Ser. Mat.*, **18**, (2010), 99-114.

[9] E. M. Elsayed, Solution and attractivity for a rational recursive sequence, *Discrete Dyn. Nat. Soc.*, Article ID 982309, 17 pages, (2011).

[10] E. M. Elsayed, On the dynamics of a higher-order rational recursive sequence, *Commun. Math. Anal.*, **12**, (2012), 117-133.

[11] J. Feuer, Periodic solutions of the Lyness max equation, *J. Math. Anal. Appl.*, **288**, (2003), 147-160.

[12] T. F. Ibrahim, On the third order rational difference equation $x_{n+1} = \frac{x_n x_{n-2}}{x_{n-1}(\alpha+\beta x_n x_{n-2})}$, *Int. J. Contemp. Math. Sci.*, **4**, (2009), 1321-1334.

[13] T. F. Ibrahim, Periodicity and analytic solution of a recursive sequence with numerical examples, *J. Interdiscip. Math.*, **12**, (2009), 701-708.

[14] T. F. Ibrahim, Solvability and attractivity of the solutions of a rational difference equation, *J. Pure Appl. Math., Adv. Appl.*, **2**, (2009), 227-237.

[15] T. F. Ibrahim, Periodicity and solution of rational recurrence relation of order six, *Applied Mathematics*, **3**, (2012), 729-733.

[16] T. F. Ibrahim, Boundedness and stability of a rational difference equation with delay, *Rev. Roum. Math. Pures Appl.*, **57**, (2012), 215-224.

[17] B. D. Iricănin and N. Touafek, On a second order max-type system of difference equations, *Indian J. Math.*, **54**, (2012), 119-142.

[18] V. L. Kocic and G. Ladas, Global behavior of nonlinear difference equations of higher order with applications, Kluwer Academic Publishers, Dordrecht, (1993).

[19] M. R. S. Kulenovic and G. Ladas, Dynamics of second order rational difference equations with open problems and conjectures, Chapman and Hall, CRC Press, (2001).

[20] V. Lakshmikantham and D. Trigiante, Theory of difference equations : Numerical methods and applications, MarcelDekker, Inc. New York-Basel, Second edition (2002).

[21] I. Ozturc, F. Bozkurt and S. Ozen, On the difference equation $y_{n+1} = \frac{\alpha+\beta e^{-y_n}}{\gamma+y_{n-1}}$, *Appl. Math. Comput.*, **181**, (2006), 1387-1393.

[22] N. Touafek and E. M. Elsayed, On the solutions of systems of rational difference equations, *Math. comput. Modelling*, **55**, (2012), 1987-1997.

[23] N. Touafek and E. M. Elsayed, On the periodicity of some systems, of nonlinear difference equations, *Bull. Math. Soc. Sci. Math. Roum., Nouv. Sr.*, **55(103)**, (2012), 217-224.

[24] N. Touafek, On a second order rational difference equation, *Hacet. J. Math. Stat.*, **41**, (2012), 867- 874.

[25] X. Li and D. Zhu, Global asymptotic stability in a rational equation, *J. Diff. Equations Appl.*, **9**, (2003), 833-839.

[26] I. Yalçinkaya, B. D. Iricanin and C. Cinar, On a max-type difference equation, *Discrete Dyn. Nat. Soc.*, Article ID 47264,(2007), 10 pages.

[27] I. Yalçinkaya, On the difference equation $x_{n+1} = \alpha + \frac{x_{n-2}}{x_n^k}$, *Fasc. Math.*, **42**, (2009), 133-139.

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