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ENDPOINT ESTIMATES IN TIME-SPACE BESOV SPACE AND APPLICATIONS TO SEMILINEAR HEAT EQUATION

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Abstract. This paper is devoted to the study of the well-posedness of semilinear heat equation with initial data in Besov space of negative regular index by using the endpoint Strichartz estimates in Time-Space Besov space. More precisely, we prove that for initial data in $B^{-\frac{2}{\alpha+2},2}_{\frac{n\alpha(\alpha+1)}{2}}$ and $\dot{B}^{-\frac{2}{\alpha+2},2}_{\frac{n\alpha(\alpha+1)}{2}}$, the semilinear heat equation has a unique local and global solution and the solutions are continuous to initial data with respect to time variable t under certain conditions of α and n. We also establish similar local results in subcritical cases for initial data in $B^{-\frac{2}{\alpha+2},r}_{p}$ under certain conditions of α, p, r and n.

Keywords. Heat equation; Semilinear; Time-Space Besov space; Bony decomposition; Well-posedness; Besov space; Semigroup.

AMS (MOS) subject classification: 35K05, 35K15, 46E35, 37C25.

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