GLOBAL DYNAMICS OF ANTI-COMPETITIVE SYSTEMS IN THE PLANE

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Abstract. We give some general results about global dynamics of an anti-competitive system of the form

$$\begin{cases} x_{n+1} &=& T_1(x_n, y_n) \\ y_{n+1} &=& T_2(x_n, y_n) \end{cases}, \quad n = 0, 1, 2, \ldots$$

where

$$T_1 : \mathcal{I} \times \mathcal{J} \to \mathcal{I}, \ T_2 : \mathcal{I} \times \mathcal{J} \to \mathcal{J} \ \text{and} \ (x_0, y_0) \in \mathcal{I} \times \mathcal{J},$$

and functions T_1 and T_2 are continuous and $T_1(x, y)$ is non-increasing in x and non-decreasing in y while $T_2(x, y)$ is non-decreasing in x and non-increasing in y. We illustrate our results by means of an example which shows a variety of typical dynamical behavior for an anti-competitive system.

Keywords. Competitive map, global stable manifold, monotonicity, period-two solution, unstable.

AMS (MOS) subject classification: Primary: 37E30, 37G99, 39A10, 39A11.

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