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NECESSARY AND SUFFICIENT CONDITIONS FOR TURNPIKE PROPERTIES OF SOLUTIONS OF OPTIMAL CONTROL SYSTEMS ARISING IN ECONOMIC DYNAMICS

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Abstract. We obtain necessary and sufficient conditions for turnpike properties of approximate solutions of nonautonomous discrete-time optimal control systems arising in economic dynamics which are determined by sequences of lower semicontinuous objective functions. To have these properties means that the approximate solutions of the problems are determined mainly by the objective functions, and are essentially independent of the choice of intervals and endpoint conditions, except in regions close to the endpoints.

Keywords. Compact metric space, good program, infinite horizon problem, minimal program, overtaking optimal program, turnpike property.

AMS (MOS) subject classification: 49J99

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