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ON THE FIRST-PASSAGE TIME PROBLEM: CUMULANT APPROACH

Fernando Ramos-Alarcón and Valeri Kontorovich

Electrical Engineering Department, Telecommunications Section, CINVESTAV-IPN.
Av. IPN 2508, Colonia San Pedro Zacatenco, C.P. 07360, Mexico City,

Corresponding author email:valeri@cinvestav.mx

Abstract. The First-Passage Time (FPT) problems have numerous practical applications in fields that go from engineering and physics to finance and biology among many others. For the case of Markov gamma processes, this paper develops a methodology that combines the first Pontryagin equation approach and the cumulant analysis in order to obtain FPT statistics. For investigation about the structure of the Probability Density Function (PDF) of the FPT an orthogonal series expansion of the distribution is made and compared with an experimental version, using Stochastic Differential Equations (SDE) simulation techniques. The results show that the FPT distribution can be accurately characterized through another gamma PDF. An application of these results is also presented.

Keywords. Markov processes, gamma processes, first-passage time statistics, cumulants, orthogonal series.

AMS (MOS) subject classification: This is optional. But please supply them whenever possible.

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