EXISTENCE OF POSITIVE SOLUTIONS FOR SINGULAR SECOND ORDER BOUNDARY VALUE PROBLEMS UNDER EIGENVALUE CRITERIA

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Abstract. We employ fixed point index properties to obtain existence results for positive solutions to the singular second order two point boundary value problem,

$$
\begin{cases}
-\frac{1}{p(t)}\left(pu'\right)'(t) = q(t)f(t, u(t)) \ \ t \in (0,1), \\
au(0) - b\lim_{t \to 0} p(t)u'(t) = 0, \\
cu(1) + d\lim_{t \to 1} p(t)u'(t) = 0,
\end{cases}
$$

where $a, b, c, d \in \mathbb{R}^+ = [0, +\infty)$, $p, q : (0,1) \to \mathbb{R}^+$ are measurable functions with $pq \in L^1_{loc}(0,1)$, q does not vanish identically on $(0,1)$ and $f : [0,1] \times \mathbb{R}^+ \to \mathbb{R}^+$ is continuous.

Keywords. Singular second order BVPs , cones, positive operator, spectral radius, fixed point index.

AMS (MOS) subject classification: 34B15, 34B16, 34B18

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