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A GENERALIZATION OF THE POINCARÉ-CARTAN INTEGRAL INVARIANT FOR A NONLINEAR NONHOLONOMIC DYNAMICAL SYSTEM

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Abstract. Based on the d'Alembert-Lagrange-Poincaré variational principle, we formulate general equations of motion for mechanical systems subject to nonlinear nonholonomic constraints, that do not involve Lagrangian undetermined multipliers. We write these equations in a canonical form called the Poincaré-Hamilton equations, and study a version of corresponding Poincaré-Cartan integral invariant which are derived by means of a type of asynchronous variation of the Poincaré variables of the problem that involve the variation of the time. As a consequence, it is shown that the invariance of a certain line integral under the motion of a mechanical system of the type considered characterizes the Poincaré-Hamilton equations as underlying equations of motion. As a special case, an invariant analogous to Poincaré linear integral invariant is obtained.

Keywords. Poincaré-Cartan integral invariant, nonlinear constraints, nonholonomic, asynchronous variation, equations of motion, Poincaré-Hamiltonian Systems.

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