Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **21 (2014)** 201-229
 http://www.watam.org

APPROXIMATING FIXED POINTS FOR LIPSCHITZIAN SEMIGROUP AND INFINITE FAMILY OF NONEXPANSIVE MAPPINGS WITH THE MEIR-KEELER TYPE CONTRACTION IN BANACH SPACES

H. Piri[1], P. Kumam[2] and K. Sitthithakerngkiet[3]

[1]Department of mathematics, University of Bonab, Bonab 55517-61167, Iran

[2]Department of Mathematics, Faculty of Science,
King Mongkut's University of Technology Thonburi (KMUTT),
Bang Mod, Thrung Khru, Bangkok 10140, Thailand

[3]Nonlinear Dynamic Analysis Research Center, Department of Mathematics, Faculty of Applied Science, King Mongkut's University of North Bangkok (KMUTNB),
Wongsawang, Bangsue, Bangkok 10800, Thailand

Abstract. In this paper we prove strong convergence theorems for approximating the fixed point of Lipschitzian semigroup and infinite family of nonexpansive mappings with respect to finite family of sequence $\{\mu_{i,n}\}_{i=1,n=1}^{m,\ \infty}$ of left strong regular invariant means and Meir-Keeler type contraction in uniformly convex and smooth Banach spaces. Our result extend and improve many recent results.

Keywords. Smooth Bacach space, Uniformly convex Bacach space, Asymptotically nonexpansive mapping, Lipschitzian semigroup, Invariant mean.

AMS (MOS) subject classification:47H09, 47H10, 47J25.

References

[1] R. P. Agarwal, D. O'Regan, and D. R. Sahu, Fixed point theory for Lipschitzian-type mappings with applications, *Topological Fixed Point Theory and Its Application*, **6**, Springer, New York, 2009.

[2] S. Atsushiba, and W. Takahashi, Strong convergence theorems for one-parameter nonexpansive semigroups with compact domains, *Fixed Point Theory and Applications*, **3**, (Y. J. Cho, J. K. Kim and S. M. Kang Eds.), pp. 15-31, Nova Science Publishers, New York, 2002.

[3] R. E. Bruck, On the convex approximation property and the asymptotic behavior of nonlinear contractions in Banach spaces, *Israel J. Math.*, **38**, (1981) 304-314.

[4] H. Brézis and F. E. Browder, Nonlinear ergodic theorem, *Bull. Amer. Math. Soc.*, **82**, (1976) 959-961.

[5] H. Brézis and F. E. Browder, Remark on nonlinear ergodic theory, *Advances in Math.*, **25**, (1977) 165-177.

[6] M. M. Day, Ameanable semigroup, *Illinois J. Math.*,**1**, (1957) 276-291.

[7] K. Eshita, H. Miyake and W. Takahashi, Strong convergence theorems for asymptotically nonexpansive mappings in general Banach spaces, Dynamics of Continuous, *Discrete and Impulsive Systems Series A: Mathematical Analysis*, **13**, (2006) 621-640.

[8] B. Halpern, Fixed points of nonexpanding maps. *Bull. Am. Math. Soc.*, **73**, (1967) 957-961.

[9] N. Hirano, K. Kido and W. Takahashi, Nonexpansive retractions and nonlinear ergodic theorems in Banach spaces, Nonlinear Anal. 28 (1988), 1269-1281.

[10] Q. Huang, L. Zhu, G. Li, Fixed point theorems for left amenable semigroups of non-Lipschitzian mappings in Banach spaces, *Fixed Point Theory and Applications* 2012, 2012:116 doi:10.1186/1687-1812-2012-116.

[11] P. Katchang and P. Kumam, A composite explicit iterative process with a viscosity method for Lipschitzian semigroup in smooth Banach space, *Bulletin of the Iranian Mathematical Society*, **37**, (2011) 143-159.

[12] K. S. Kim, nonlinear ergodic theorems of nonexpansive type mappings, *J. math. Anal. Appl.*, **358**, (2009) 261-272.

[13] K. S. Kim, Convergence of a hybrid algorithm for a reversible semigroup of nonlinear operators in banach spaces, *Nonlinear Analysis*, **73**, (2010), no. 3, 3413-3419.

[14] W. R. Mann, Mean value methods in iteration. *Proc. Am. Math. Soc.*, **4**,(1953) 506-510.

[15] A. T. Lau, H. Miyake, and W. Takahashi, Approximation of fixed points for amenable semigroups of nonexpansive mappings in Banach spaces, *Nonlinear Anal.*, **67**, (2007), no. 4, 1211-1225.

[16] A. T. Lau, N. Shioji, and W. Takahashi, Existence of nonexpansive retractions for amenable semigroups of nonexpansive mappings and nonlinear ergodic theorems in Banach spaces, *J. Funct. Anal.*, **161**, (1999), no. 1, 62-75.

[17] G. G. Lorentz, a contribution to the theory of divrgent series, *Acta Math.*, **80**, (1948) 167-190.

[18] H. Miyake and W. Takahashi, Nonlinear mean ergodic theorems for nonexpansive semigroup in Banach spaces. *J. fixed point Theory and Appl.*, **2**, (2007) 360-382.

[19] T. C. Lim, On characterizations of Meir-Keeler contractive maps. *Nonlinear Anal.*, **46**, (2001) 113-120.

[20] A. Petrusel and J.C. Yao, Viscosity approximation to common fixed points of families of nonexpansive mappings with generalized contractions mappings. *Nonlinear Anal.*, **69**, (2008), 1100-1111.

[21] H. Piri, Hybrid pseudo-viscosity approximation schemes for systems of equilibrium problems and fixed point problems of infinite family and semigroup of non-expansive mappings. *Nonlinear Anal.*, **74**, (2011) 6788–6804.

[22] H. Piri and A. H. Badali, Strong convergence theorem for amenable semigroups of nonexpansive mappings and variational inequalities. *Fixed Point Theory Appl.*, 2011 (2011)

[23] H. Piri and P. Kumam, Approximating fixed points for a reversible semigroup of Lipschitzian mappings in a smooth Banach space, *Journal of Inequalities and Applications*, 2013, 2013:555

[24] S. Reich, Asymptotic behavior of contraction in Banach spaces, J. Math. Anal. Appl. 44 (1973) 57-70.

[25] S. Reich, Nonlinear evolution equetions and nonlinear ergodic theorems, *Nonlinear Analysis.*, **1**, (1977) 319-330.

[26] S. Reich, almost convergence and nonlinear ergodic theorems, *J. Approx. Theory.*, **24**, (1978) 269-272.

[27] S. Reich and A.J. Zaslavski, Convergence of Krasnoselskii-Mann iterations of nonexpansive operators. *Math. Comput. Model.*, **32**, (2000) 1423-1431.

[28] S. Saeidi, Approximating common fixed points of Lipschitzian semigroup in smooth Banach spaces, *Fixed Point Theory and Applications.* 2008, Article ID 363257, 17 pages.

[29] S. Saewan and P. Kumam, Explicit iterations for Lipschitzian semigroups with the Meir-Keeler type contraction in Banach spaces, *Journal of Inequalities and Applications* 2012, 2012:279

[30] K. Shimoji and W. Takahashi, Strong convergence to common fixed points of infinite nonexpansive mappings and applications, *Taiwanese J. Math.*, **5**, (2001) 387-404.

[31] W. Takahashi, *Nonlinear functional analysis,Fixed point theory and its applications*, Yokohama Publishers, Yokohama, 2000.

[32] K. Wattanawitoon and P. Kumam, Strong Convergence to Common Fixed Points for Countable Families of Asymptotically Nonexpansive Mappings and Semigroups, *Fixed Point Theory and Applications* (2010), Article ID 301868, 15 pages doi:10.1155/2010/301868

[33] H. K. Xu, An iterative approach to quadratic optimization, *J. Optim. Theory Appl.*, **116**, (2003), no. 3, 659-678.

[34] S. S. Zhang, L. Yang and J. A. Liu, Strong convergence theorems for nonexpansive mappings in Banach spaces, *Appl. Math. mech.* (English Ed.) **28** (2007) 1287-1297.

Received January 2014; revised April 2014.

email: journal@monotone.uwaterloo.ca
http://monotone.uwaterloo.ca/∼journal/