PRICING VARIANCE SWAPS WITH STOCHASTIC VOLATILITY UNDER JUMP-DIFFUSION*

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Abstract. This article discusses the valuation of discretely sampled variance swaps within the frame of Heston's two-factor stochastic volatility under jump-diffusion model. As a main contribution of this paper, we prove the independence of a Brownian motion and a compound Poisson process so that the realized variance can be decomposed into two parts. Furthermore, we not only introduce the Merton's jump-diffusion model with jump sizes normal distributed as well as S. G. Kou's model with jump sizes double exponentially distributed, but also find a closed-form solution in both circumstances based on the work done by Song-Ping Zhu and Guang-Hua Lian. In the end, a numerical example is analyzed to illustrate the theoretical results.

Keywords. Variance swaps, stochastic volatility, jump-diffusion, double exponential distribution, continuous dividend rate

AMS subject classification: 35QXX

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