Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **21 (2014)** 317-331
Copyright ©2014 Watam Press http://www.watam.org

DYNAMICS OF THE RATIONAL DIFFERENCE EQUATION

$$x_{n+1} = Ax_n + Bx_{n-k} + Cx_{n-l} + \frac{bx_nx_{n-k}x_{n-l}}{dx_{n-k} - ex_{n-l}}$$

M. A. El-Moneam[1] and E. M. E. Zayed[2]

[1]Mathematics Department, Faculty of Science and Arts in Farasan,
Jazan University, Kingdom of Saudi Arabia.
mabdelmeneam2004@yahoo.com

[2]Mathematics Department, Faculty of Science,
Zagazig University, Zagazig, Egypt.
e.m.e.zayed@hotmail.com

Abstract. In this article, we study the periodicity, the boundedness and the global stability of the positive solutions of the following nonlinear difference equation

$$x_{n+1} = Ax_n + Bx_{n-k} + Cx_{n-l} + \frac{bx_nx_{n-k}x_{n-l}}{dx_{n-k} - ex_{n-l}}, \qquad n = 0, 1, 2, ...$$

where the coefficients $A, B, C, b, d, e \in (0, \infty)$, while k and l are positive integers. The initial conditions $x_{-l}, ..., x_{-k}, ..., x_{-1}, x_0$ are arbitrary positive real numbers such that $k < l$. Some numerical examples will be given to illustrate our results.

Keywords. Difference equations, prime period two solution, boundedness character, locally asymptotically stable, global attractor, global stability.

AMS (MOS) subject classification: 39A10,39A11,39A99,34C99.

References

[1] R. Abu-Saris, C. Cinar and I. Yalcinkaya, On the asymptotic stability of $x_{n+1} = (a+x_n x_{n-k})/(x_n + x_{n-k})$, Computers & Mathematics with Applications, 56 (2008), 1172-1175.

[2] R. Agarwal, Difference Equations and Inequalities. Theory, Methods and Applications, Vol.228, Marcel Dekker Inc, New York, Second Edition 2000.

[3] S. N. Elaydi, An introduction to difference equations, Springer, New York, 1999.

[4] E. M. Elabbasy, H. El- Metwally and E. M. Elsayed, On the difference equation $x_{n+1} = ax_n - bx_n/\left(cx_n - dx_{n-1}\right)$, Advances in Difference Equations, Volume 2006, Article ID 82579, pages 1-10, doi: 10.1155/2006/82579.

[5] M. A. El-Moneam, On the dynamics of the higher order nonlinear rational difference equation, Math. Sci. Lett. 3, No. 2(2014), 121-129.

[6] M. A. El-Moneam and S.O. Alamoudy, On study of the asymptotic behavior of some rational difference equations, DCDIS Series A: Mathematical Analysis, 21(2014), 89-109.

[7] M. A. El-Moneam and E. M. E. Zayed, Dynamics of the rational difference equation, Inf. Sci. Lett. 3, No. 2(2014), 1-9.

[8] M. E. Erdogan, C. Cinar and I. Yalcinkaya, On the dynamics of the recursive sequence, Computers & Mathematics with Applications, 61(2011), 533-537.

[9] H. M. EL- Owaidy, A. M. Ahmed and Z. Elsady, Global attractivity of the recursive sequence $x_{n+1} = (\alpha - \beta x_{n-k})/(\gamma + x_n)$, J. Appl. Math. & Computing, 16(2004), 243-249.

[10] E. A. Grove and G. Ladas, Periodicities in nonlinear difference equations, Vol.4, Chapman & Hall / CRC, 2005.

[11] G. Karakostas and S. Stevic′, On the recursive sequences $x_{n+1} = A + f(x_n, ..., x_{n-k+1})/x_{n-1}$, Comm. Appl. Nonlinear Analysis, 11(2004), 87-100.

[12] W. G. Kelley and A. C. Peterson, Difference equations: An introduction with applications, Harcour Academic, New York, 2001.

[13] V. L. Kocic and G. Ladas, Global behavior of nonlinear difference equations of higher order with applications, Kluwer Academic Publishers, Dordrecht, 1993.

[14] M. R. S. Kulenovic and G. Ladas, Dynamics of second order rational difference equations with open problems and conjectures, Chapman & Hall / CRC, Florida, 2002.

[15] T. I. Saary, Modern nonlinear equations, Mc Graw Hill, New York, 1967.

[16] M. Saleh and S. Abu-Baha, Dynamics of a higher order rational difference equation, Appl. Math. Comput., 181(2006), 84-102.

[17] X. Yang, B. Chen, G. M. Megson and D. J. Evans, Global attractivity in a recursive sequence, Appl. Math. Comput., 158(2004), 667-682.

[18] E. M. E. Zayed and M. A. El-Moneam, On the rational recursive sequence $x_{n+1} = (D + \alpha x_n + \beta x_{n-1} + \gamma x_{n-2})/(Ax_n + Bx_{n-1} + Cx_{n-2})$, Comm. Appl. Nonl. Anal., 12(2005), 15-28.

[19] E. M. E. Zayed and M. A. El-Moneam, On the rational recursive sequence $x_{n+1} = (\alpha x_n + \beta x_{n-1} + \gamma x_{n-2} + \delta x_{n-3})/(Ax_n + Bx_{n-1} + Cx_{n-2} + Dx_{n-3})$, J. Appl. Math. & Computing, 22(2006), 247-262.

[20] E. M. E. Zayed and M. A. El-Moneam, On the rational recursive sequence $x_{n+1} = \left(A + \sum_{i=0}^{k} \alpha_i x_{n-i}\right) / \sum_{i=0}^{k} \beta_i x_{n-i}$, Math. Bohemica, 133(2008), No.3, 225-239.

[21] E. M. E. Zayed and M. A. El-Moneam, On the rational recursive sequence $x_{n+1} = \left(A + \sum_{i=0}^{k} \alpha_i x_{n-i}\right) / \left(B + \sum_{i=0}^{k} \beta_i x_{n-i}\right)$, Int. J. Math. & Math. Sci., Volume 2007, Article ID 23618, 12 pages, doi: 10.1155/2007/23618.

[22] E. M. E. Zayed and M. A. El-Moneam, On the rational recursive sequence $x_{n+1} = ax_n - bx_n/\left(cx_n - dx_{n-k}\right)$, Comm. Appl. Nonlinear Analysis, 15(2008), 47-57.

[23] E. M. E. Zayed and M. A. El-Moneam, On the Rational Recursive Sequence $x_{n+1} = \left(\alpha + \beta x_{n-k}\right)/\left(\gamma - x_n\right)$, J. Appl. Math. & Computing, 31(2009) 229-237.

[24] E. M. E. Zayed and M. A. El-Moneam, On the rational recursive sequence $x_{n+1} = Ax_n + \left(\beta x_n + \gamma x_{n-k}\right)/\left(Cx_n + Dx_{n-k}\right)$, Comm. Appl. Nonl. Anal., 16(2009), 91-106.

[25] E. M. E. Zayed and M. A. El-Moneam, On the Rational Recursive Sequence $x_{n+1} = \gamma x_{n-k} + \left(ax_n + bx_{n-k}\right)/\left(cx_n - dx_{n-k}\right)$, Bull. Iranian Math. Soc., 36(2010) 103-115.

[26] E. M. E. Zayed and M. A. El-Moneam, On the global attractivity of two nonlinear difference equations, J. Math. Sci., 177(2011), 487-499.

[27] E. M. E. Zayed and M. A. El-Moneam, On the rational recursive two sequences $x_{n+1} = ax_{n-k} + bx_{n-k}/\left(cx_n + \delta dx_{n-k}\right)$, Acta Math. Vietnamica, 35(2010), 355-369.

[28] E. M. E. Zayed and M. A. El-Moneam, On the rational recursive sequence $x_{n+1} = Ax_n + Bx_{n-k} + \left(\beta x_n + \gamma x_{n-k}\right)/\left(Cx_n + Dx_{n-k}\right)$, Acta Appl. Math., 111(2010), 287-301.

[29] E. M. E. Zayed and M. A. El-Moneam, On the rational recursive sequence $x_{n+1} = \left(\alpha_0 x_n + \alpha_1 x_{n-l} + \alpha_2 x_{n-k}\right)/\left(\beta_0 x_n + \beta_1 x_{n-l} + \beta_2 x_{n-k}\right)$, Math. Bohemica, 135(2010), 319-336.

[30] E. M. E. Zayed and M. A. El-Moneam, On the rational recursive sequence $x_{n+1} = \left(A + \alpha_0 x_n + \alpha_1 x_{n-\sigma}\right)/\left(B + \beta_0 x_n + \beta_1 x_{n-\tau}\right)$, Acta Math. Vietnamica, 36(2011), 73-87.

[31] E. M. E. Zayed and M. A. El-Moneam, On the global asymptotic stability for a rational recursive sequence, Iranian Journal of science and Technology (A: siences), 35 (A4) (2011), 333-339.

[32] E. M. E. Zayed and M. A. El-Moneam, On the rational recursive sequence $x_{n+1} = \frac{\alpha_0 x_n + \alpha_1 x_{n-l} + \alpha_2 x_{n-m} + \alpha_3 x_{n-k}}{\beta_0 x_n + \beta_1 x_{n-l} + \beta_2 x_{n-m} + \beta_3 x_{n-k}}$, WSEAS Transactions Math.,11(2012), 373-382.

[33] E. M. E. Zayed and M. A. El-Moneam, On the qualitative study of the nonlinear difference equation $x_{n+1} = \frac{\alpha x_{n-\sigma}}{\beta + \gamma x_{n-\tau}^p}$, Fasciculi Mathematici, 50(2013), 137-147.

[34] E. M. E. Zayed and M. A. El-Moneam, Dynamics of the rational difference equation $x_{n+1} = \gamma x_n + \frac{\alpha x_{n-l} + \beta x_{n-k}}{A x_{n-l} + B x_{n-k}}$, Comm. Appl. Nonl. Anal., 21(2014), 43-53.

Received February 2014; revised August 2014.

email: journal@monotone.uwaterloo.ca
http://monotone.uwaterloo.ca/∼journal/