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SOLVING A CLASS OF THREE-ORDER MAX-TYPE DIFFERENCE EQUATIONS

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Abstract.
In this paper we will investigate the closed form solution of a class of max-type difference equation in the form

$$x_{n+1} = max\{\frac{A}{x_n^k}, x_{n-2}^k\}, \qquad n \in \mathbb{N}_0$$

where $k \geq 1$, $A \geq 1$ and the initial conditions x_{-2}, x_{-1}, x_0 are positive real numbers.Some other cases are given when $A \leq 0$. The results in this paper generalize some results for Elsayed and Stevic in [7].

Keywords. Difference equation,periodicity, eventually periodic, solutions.

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