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BICOMPLEX VERSION OF STIELTJES TRANSFORM AND APPLICATIONS

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Abstract. Motivated by the recent developments of the integral transform in bicomplex form, in this paper we derive the formula for bicomplex version of Stieltjes transform, it's inverse and relationship with bicomplex Laplace transform. We have discussed some of it's basic operational properties and convolution theorem. Applications of bicomplex Stieltjes transform in finding the solution of singular integral equation, probability distribution theory and spectral analysis of random matrices in signal processing are given.

Keywords. Bicomplex Number, bicomplex Laplace transform, Stieltjes transform.

AMS (MOS) subject classification: Primary 30G35; Secondary 42B10.

References

[1] A. Banerjee, S.K. Datta and A. Hoque, Fourier transform and its inverse for functions of bicomplex variables, *arXiv.org math arXiv:1404.4236*, (2014) 1-20.

[2] A. Banerjee, S.K. Datta and A. Hoque, Inverse Laplace transform for Bi-complex variables, *Mathematical inverse problems*, **1(1)**, (2014) 8-24.

[3] A. Charabarti and S.C. Martha, Method of solution of singular integral equations, *Mathematical Science a Springer open journal*, **6(7)**, (2012) 1-29.

[4] A. Erdelyi, W. Magnus, F. Oberhettinger and F.G. Tricomi, Tables of Integral transforms, McGraw-Hill, New York, Londan and Toronto, **2**, 1954.

[5] A. Kumar and P. Kumar, Bicomplex version of Laplace Transform, *International Journal of Engg. and Tech.*, **3(3)**, (2011) 225-232.

[6] A. Nica and R. Speicher, Lectures on the Combinatorics of Free Probability, London Mathematical Society Lecture Note Series 365, 2006.

[7] B. Cakmak, Non-Hermitian Random Matrix Theory for MIMO Channels, NTNU-Trondheim Norwegian University of sci. and Technology 2012.

[8] C. Segre, Le rappresentazioni reale delle forme complessee Gli Enti Iperalgebrici, *Math. Ann.*, **40**, (1892) 413-467.

[9] D. Alpay, M.E.L. Elizarraras, M. Shapiro and Struppa D.C., Basics of functional analysis with bicomplex scalars, and bicomplex Schur analysis, *arXiv:1304.0781V1[math.CV]*, (2013) 1-106.

[10] D.V. Widder, An introduction to transform theory, Academic press, New-York and London 1971.

[11] D.V. Widder, The Stieltjes Transform, *Transactions of the American Mathematical Society,* **43**, (1938) 7-60.

[12] E.P. Wigner, Characteristic Vectors of Bordered Matrices with Infinite Dimensions, *Ann. of Math.*, **62**,(1955) 548-564.

[13] G. B. Price, An Introduction to multicomplex spaces and functions, Marcel Dekker Inc., New York, 1991.

[14] G.S. Dragoni, Sulle funzioni olomorfe di una variable bicomplessa, *Reale Acad. d'Italia Mem. Class Sci. Fic. Mat. Nat.*, **5**, (1934) 597-665.

[15] H.M. Srivastava and V.K. Tuan, A new convolution theorem for the Stieltjes transform and it's application to a class of singular integral equations, *Arch. Math. Birkhauser verlag basel,* **64**, (1995) 144-149.

[16] L. Debnath and D. Bhatta, Integral transforms and their applications, Chaman & Hall/CRC Taylor & Francis Group, ISBN 1-58488-575-0, 2006.

[17] M. Futagawa, On the theory of functions of quaternary variable-I, *Tohoku Math. J.*, **29**, (1928) 175-222.

[18] M. Futagawa, On the theory of functions of quaternary variable-II, *Tohoku Math. J.*, **35**, (1932) 69-120.

[19] R. Agarwal, M.P. Goswami and R.P. Agarwal, Convolution Theorem and Applications of Bicomplex Laplace Transform, *Advances in Mathematical Sciences and Applications*, **24**, (2014) (in press).

[20] R. Couillet and M. Debbah, Signal Processing in Large Systems: A New Paradigm, *IEEE Signal Process. Mag.*, **30(1)**, (2013) 24-39.

[21] R.R. Müller and B. Cakmak, Channel Modelling of MU-MIMO Systems by Quaternionic Free Probability, *IEEE International Symposium on Information Theory Proceedings (ISIT)*, (2012) 2656 - 2660.

[22] S. Choi, The Inversion formula of the Stieltjes transform of spectral distribution, *Journal of the Chungcheong Mathematical Society,* **22(3)**, (2009) 519-524.

[23] S.K. Sinha, On two new characterizations of Stieltjes transforms for distributions, *International J. of Math. and Math. Sci.*, **8(4)**, (1985) 719-723.

[24] S. Rönn, Bicomplex algebra and function theory, *arXiv:math/0101200v1 [math.CV]*, (2001) 1-71.

[25] Y. Huang, The Stieltjes Transforms of Symmetric probability distribution functions, Master thesis, National Sun Yat-sen University, 2007.

[26] Z. Bai and J.W. Silverstien, Spectral analysis of Large dimensional Random Matrices, Springer Second edition, ISBN 978-1-4419-0660-1, 2010.

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