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STABILITY PROPERTIES OF A DELAYED HIV DYNAMICS MODEL WITH BEDDINGTON-DEANGELIS FUNCTIONAL RESPONSE AND ABSORPTION EFFECT

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Abstract. In this paper, stability properties of a class of HIV virus infection model with Beddington-DeAngelis functional response and absorption effect are investigated. Our mathematical analysis shows that stability properties are completely determined by the basic reproduction number R_0 of the model. Utilizing characteristic equation of the model, we established that the infection free equilibrium and the chronic infection equilibrium is locally asymptotically stable if $R_0 \leq 1$ and $R_0 > 1$, respectively. By means of Liapunov functionals and LaSalle's invariance principle, it is derived that, if $R_0 \leq 1$, the infection free equilibrium is globally asymptotically stable. Moreover, the numerical simulations are also carried out in order to illustrate the theoretical results.

Keywords. Virus model; Absorption effect; Liapunov functional; Global stability.

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