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COMPLEX DYNAMICS OF FORCED LSTAR MODEL WITH DELAY

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Abstract. In this paper, the dynamical behaviour of forced LSTAR model with delay is considered for different levels of noise intensity. The existence and stability of the equilibria of the deterministic system are studied. Numerical simulations are employed to show the model's complex dynamics by means of the largest Lyapunov exponents, bifurcations, time series diagrams and phase portraits. The phenomena of noise-induced intermittency is also discussed.

Keywords. Nonlinear autoregressive models; Forced LSTAR model; Bifurcations; Neimark-Sacker bifurcation; Chaos; Noise-induced chaos.

AMS (MOS) subject classification: 34K60; 34K23; 37M10; 91B70; 93E03.

References

[1] D. Bacon and D. Watts, Estimating the transition between two intersecting straight lines, *Biometrika*, **58 (3)**, (1971) 525-534.

[2] J. Balagtas and M. Holt, The commodity terms of trade, unit roots, and nonlinear alternatives: a smooth transition approach, *American Journal of Agricultural Economics*, **91 (1)**, (2009) 87-105.

[3] I. Bashkirtseva and L. Ryashko, Stochastic sensitivity analysis of noise-induced intermittency and transition to chaos in one-dimensional discrete-time systems, *Physica A*, **392**, (2013) 295-306.

[4] J. H. E. Cartwright, Nonlinear stiffness, Lyapunov exponents, and attractor dimension, *Phys. Lett. A*, **264**, (1999) 298-304.

[5] L. A. Craig and M. T. Holt, Mechanical refrigeration, seasonality, and the hogecorn cycle in the United States: 1870-1940, *Explorations in Economic History*, **45 (1)**, (2008) 30-50.

[6] K. Chan and H. Tong, On estimating thresholds in autoregressive models, *Journal of Time Series Analysis*, **7 (3)**, (1986) 179-190.

[7] S. Chatterjee and M. Yilmaz, Chaos, fractals and statistics, *Statistical Science*, **7**, (1992) 49-68.

[8] S. Chen and S.A Billings, Modeling and analysis of non-linear time series, *Internat. J. Control*, **50 (6)**, (1989) 2151-2171.

[9] R. L. Devaney, An Introduction to Chaotic Dynamical Systems, Addison-Wesely publishing, New York 1989.

[10] O. Eitrheim and T. Terasvirta, Testing the adequacy of smooth transition autoregressive models, *Journal of Econometrics*, **74 (1)**, (1996) 59-75.

[11] S. N. Elaydi, An Introduction to Difference Equations, Springer-Verlag Publishers, New York, 2005.

[12] A. Elhassanein, Complex dynamics of logistic self-exciting threshold autoregressive model, *J. Comput. Theor. Nanosci.*, **12 (4)**, (2015) 1-7.

[13] A. Elhassanein, Complex dynamics of a forced discretized version of the Mackey-Glass delay differential equation, *Discrete Contin. Dyn. Syst. Ser. B*, **20 (1)**, (2015) 93-105.

[14] A. Elhassanein, Complex dynamics of a stochastic discrete modified Leslie-Gower predator-prey model with Michaelis-Menten type prey harvesting, *Computational Ecology and Software*, **4 (2)**, (2014) 116-128.

[15] A. Elhassanein, On the control of forced process feedback nonlinear autoregressive model, *J. Comput. Theor. Nanosci.*, Accpted.

[16] P. H. Franses and D. van Dijk, Nonlinear Time Series Models in Empirical Finance, Cambridge University Press, Cambridge, 2003.

[17] M. A. Ghazal and A. Elhassanein, Dynamics of EXPAR models for high frequency data, *Int. J. Appl. Math. Stat.*, **14**, (2009), 88-96.

[18] C. W. Granger and A.P. Andersen, An Introduction to Bilinear Time Series Models, Vanderhoek and Ruprecht, Gottingen, 1978.

[19] D. Gulick, Encounters with Chaos, McGraw Hill, New York, 1992.

[20] A. Hall, J. Skalin and T. Terasvirta, A nonlinear time series model of El Nino, *Environ. Model. and Softw*, **16 (2)**, (2001) 139-146.

[21] J. L. Kaplan and Y.A Yorke, A regime observed in a fluid flow model of Lorenz, *Comm. Math. Phys.*, **67**, (1979) 93-108.

[22] H.-W Lorenz, Nonlinear Dynamical Economics and Chaotic Motion, Lectures Notes in Economics and Mathematical Systems 334, Springer, Berlin, 1993.

[23] R. Luukkonen, P.Saikkonen and T. Terasvirta, Testing linearity against smooth transition autoregressive models, *Biometrika*, **75 (3)**, (1988) 491- 499.

[24] T. Ozaki and H. Oda, Non-linear Time Series Model Identification by Akaike's Information Criterion, In:Information and Systems(B. Dubuisson, ed.), Prgamon Press, Oxford 1978.

[25] T. Puu, Attractors, Bifurcations and Chaos: Nonlinear Phenomena in Economics, Springer, Berlin, 2000.

[26] N. Sarantis, Modeling non-linearities in real effective exchange rates, *J. Int. Money and Finance*, **18 (1)**, (1999) 27-45.

[27] J. Skalin and T. Terasvirta, Modeling asymmetries and moving equilibria in unemployment rates, *Macroecon. Dyn.*, **6 (2)**, (2002) 202-241.

[28] H. S. Steven, Nonlinear Dynamics and Chaos: with Applications to Physics, Biology, Chemistry and Engineering, Perseus Books Publishing, L. L. C., Massachusetts 1994.

[29] T. Terasvirta and H. Anderson, Characterizing nonlinearities in business cycles using smooth transition autoregressive models, *Journal of Applied Econometrics*, **7**, (1992) S119-S136.

[30] T. Terasvirta, Specification, estimation, and evaluation of smooth transition autoregressive models, *Journal of the American Statistical Association*, **89 (425)**, (1994) 208-218.

[31] T. Terasvirta, Modelling nonlinearity in US Gross national product 1889-1987, *Empirical Economics*, **20 (4)**, (1995) 577-597.

[32] T. Terasvirta, D. Tjostheim and C. W. J. Granger, Modelling Nonlinear Economic Time Series, Number 9780199587155 in OUP Catalogue, Oxford University Press, Oxford December 2010.

[33] H. Tong, Threshold models in non-linear time series analysis,Lecture Notes in Statistics No. 21, Springer, Heidelberg, 1983.

[34] H. Tong, Non-linear Time Series: A Dynamical System Approach, Oxford University Press, Oxford 1990.

[35] H. Tong, On a threshold model, in C.H. Chen (ed.), Pattern Recognition and Signal Processing, Amsterdam: Sijthoff & Noordhoff, (1978) 101-141.

[36] H. Tong and K. S. Lim, Threshold autoregressions, limit cycles, and data, *J. Roy. Statist. Soc. B*, **42**, (1980) 245-292.

[37] D. Ubilava, Modeling nonlinearities in the U.S. soybean-to-corn price ratio: a smooth transition autoregression approach, *Agribusiness: An International Journal*, **28 (1)**, (2012) 29-41.

[38] D. Ubilava and C. G. Helmers, Forecasting ENSO with a smooth transition autoregressive model, *Environmental Modelling & Software*, **40**, (2013) 181-190.

[39] D. Ubilava, El Nino, La Nina, and world coffee price dynamics, *Agricultural Economics*, **43 (1)**, (2012) 17-26.

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