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DYNAMIC MODELLING AND NONLINEAR CONTROL OF A FRONT-WHEEL-DRIVE VEHICLE SUBJECT TO HOLONOMIC AND NONHOLONOMIC CONSTRAINTS

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Abstract. This work considers the problem of dynamic modelling and nonlinear control of a front-wheel-drive vehicle, whose motion is subject to holonomic and nonholonomic velocity constraints. The mechanical configuration of the vehicle results in a generalized steering wheel torque associated with the turning angle of the steering wheel, and a generalized drive system torque associated with the rotational angle of the drive system. The steering wheel steers the front wheels via a steering mechanism leading to nonlinear geometric constraints. Furthermore, the drive system drives the front wheels via a differential gear-box and side-shafts. It is assumed that all four wheels of the vehicle roll without slipping resulting in nonholonomic velocity constraints. Thus, the vehicle subsystems lead to a set of holonomic and nonholonomic velocity constraints which are not independent. In this work, the constraints are not reduced to a set of independent velocity constraints. The original form and structure of the constraints are preserved. A methodology based on Lagrangian mechanics is developed and applied to derive the vehicle kinematic and dynamic models using all the velocity constraints. In addition, a nonlinear feedback control strategy is derived for the generalized steering wheel and drive system torques such that the vehicle steering wheel turning angle, and the drive system rotational velocity asymptotically track specified reference trajectories, respectively. The constrained motion of the controlled vehicle dynamic model is computed, and used to obtain the vector of generalized constraint forces, and then the vector of Lagrange multipliers by applying the Moore-Penrose generalized inverse.

Keywords. Front-wheel-drive vehicle, Differential gear-box, Front wheel steering mechanism, Instantaneous center of rotation, Nonlinear control, Geometric constraints, Nonholonomic and holonomic velocity constraints, Independent and not independent velocity constraints, Lagrange equations, Generalized applied forces, Generalized constraint forces, Kinematic model, Reduced dynamic model, Lagrange multipliers, Moore-Penrose generalized inverse.

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