LIFE SPAN AND LARGE TIME BEHAVIOR OF SOLUTIONS TO A DEGENERATE PARABOLIC EQUATION WITH WEIGHTED SOURCE

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Abstract. This paper studies the Cauchy problem of a degenerate parabolic equation not in divergence form with weighted source $u_t = u^p \Delta u + a(x)u^q$ in $\mathbb{R}^n \times (0, T)$, where $p \geq 1$, $q > 1$, and the positive weight function $a(x)$ is of the order $|x|^m$ with $m > 0$. In a previous paper of the authors [Critical exponents in a degenerate parabolic equation with weighted source, Applicable Analysis 92 (2013), 814–830], the the critical and the second critical exponents were determined as $q_c = p+1$ and $a^* = \frac{2+m}{q-p-1}$ respectively. Now we continue to consider the life span for the blow-up solutions and the large time behavior for the global solutions in the subregion of the coexistence exponent region with $q > p + 1 + \frac{2+m}{N} > q_c$, or equivalently, $0 < a^* < N$.

Keywords. Non-divergent parabolic equation; Life span; Large time behavior; Critical exponents

AMS (MOS) subject classification: 35K55; 35B40; 35B33

1   Introduction

In this paper, we consider the nonlinear diffusion equation not in divergence form with weighted source

$$\begin{cases} u_t = u^p \Delta u + a(x)u^q, & (x,t) \in \mathbb{R}^N \times (0, T), \\ u(x,0) = u_0(x), & x \in \mathbb{R}^N, \end{cases} \tag{1}$$

where $p \geq 1$, $q > 1$, $0 < u_0 \in C^0(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$, and $0 < a(x) \in C^0(\mathbb{R}^N)$, $a(x) \sim |x|^m$ in the meaning

$$c_1|x|^m \leq a(x) \leq c_2|x|^m, \quad x \in \mathbb{R}^N \tag{2}$$

with $c_1, c_2, m > 0$.