HADAMARD FACTORIZATION OF HURWITZ STABLE POLYNOMIALS

Carlos Arturo Loredo-Villalobos[1] and Baltazar Aguirre-Hernández[1]

[1]Departamento de Matemáticas
Universidad Autónoma Metropolitana
Av. San Rafael Atlixco 186, Col. Vicentina, Iztapalapa 09340, México, D.F, México.

Abstract. The Hurwitz stable polynomials are important in the study of differential equations systems and control theory (see [7] and [19]). A property of these polynomials is related to Hadamard product. Consider two polynomials $p, q \in R[x]$:

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

$$q(x) = b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0$$

the Hadamard product $(p * q)$ is defined as

$$(p * q)(x) = a_k b_k x^k + a_{k-1} b_{k-1} x^{k-1} + \cdots + a_1 b_1 x + a_0 b_0$$

where $k = \min(m, n)$. Some results (see [16]) shows that if $p, q \in R[x]$ are stable polynomials then $(p * q)$ is stable, also, *i.e.* the Hadamard product is closed; however, the reciprocal is not always true, that is, not all stable polynomial has a factorization into two stable polynomials the same degree n, if $n \geq 4$ (see [15]). In this work we will give some conditions to Hadamard factorization existence for stable polynomials.

Keywords. Hurwitz polynomial, Hadamard product, Hadamard stable factorization.

AMS (MOS) subject classification: 93D99

1   Introduction

It is known that the problem of finding conditions for verifying if a given polynomial has all of its roots with negative real part was proposed by Maxwell [26]. The polynomials with such property are named stable (Hurwitz) polynomials. Interesting information about Hurwitz polynomials can be found in [13] and [22]. The Routh-Hurwitz criterion [20], the Hermite-Biehler theorem [17] and Stability test (see [7]) are perhaps the most famous criteria. Due to the presence of uncertainties when a physical phenomenon is modeled, we must often study the stability of families of polynomials. Excellent references about families of Hurwitz polynomials are [1], [6] and [7]. With respect to this subject it is important to mention Kharitonov's theorem [21], which is the most famous result about families of stable polynomials. Kharitonov studied the stability of interval families. Other questions about interval polynomials can be consulted in [9], [10] and [29]. Since the set of Hurwitz polynomials is not convex, the stability of segments of polynomials