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SEPARATION FOR AN ELLIPTIC DIFFERENTIAL OPERATOR IN A WEIGHTED HILBERT SPACE WITH ITS APPLICATION TO AN EXISTENCE AND UNIQUENESS THEOREM

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Abstract. In this paper, we have studied the separation for the general elliptic differential operator

$$Au(x) = -\sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial^2 u(x)}{\partial x_i \partial x_j} + \sum_{j=1}^{n} b_j(x)\frac{\partial u(x)}{\partial x_j} + V(x)u(x),$$

for all $x \in R^n$, in the weighted Hilbert space $H = L_{2,k}(R^n, H_1)$ with a positive weight function $k(x) \in C^1(R^n)$ and the operator potential $V(x) \in C^2(R^n, L(H_1))$, where $L(H_1)$ is the space of all bounded linear operators on the Hilbert space H_1. Here the coefficients $a_{ij}(x) \in C^2(R^n)$ and $b_j(x) \in C^1(R^n)$ are real positive functions. Moreover, we have studied the existence and uniqueness Theorem for the solution of the nonhomogeneous elliptic differential equation $Au(x) = f(x)$ in the weighted Hilbert space H where $f(x) \in H$ as an application of the separation approach.

Keywords. Separation, General elliptic operator, Operator potential, Weighted Hilbert space, Weight function, Coercive estimate, Existence and uniqueness Theorem.

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1 Introduction

The concept of separation for differential operators was first introduced by Everitt and Giertz [10,11]. They have obtained the separation results for the Sturm Liouville differential operator

$$Au(x) = -u''(x) + V(x)u(x),\ x \in R, \tag{1}$$

in the space $L_2(R)$. They have studied the following question: What are the conditions on $V(x)$ such that if $u(x) \in L_2(R)$ and $Au(x) \in L_2(R)$ imply that both of $u''(x)$ and $V(x)u(x) \in L_2(R)$? More fundamental results of separation of differential operators were obtained by Everitt and Giertz [12,13]. A number of results concerning the property referred to the separation of differential operators was discussed by Biomatov [6], Otelbaev [21], Zettle [30] and Mohamed etal [14-19]. The separation for the differential operators with the matrix potential was first studied by Bergbaev [5]. Brown [7] has shown that certain properties of positive solutions of disconjugate second