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A FINITE DIFFERENCE METHOD FOR THE FITZHUGH-NAGUMO EQUATIONS

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Abstract. In this paper, we propose a special finite difference method to approximate traveling wave solutions of the FitzHugh-Nagumo equations. Consistency and stability of the method have been investigated. Numerical results are provided to illustrate the performance of the method. The threshold phenomenon of the neural system have also been studied numerically.

Keywords. FitzHugh-Nagumo equations; finite difference method; nonlinear reaction-diffusion system; traveling wave solution; threshold phenomenon

AMS (MOS) subject classification: 65M06; 35K55

1 Introduction

The study of neuroscience started from 1950s after Hodgkin and Huxley's famous model on action potential propagation in the giant axon of the Atlantic squid [11]. The Hodgkin-Huxley (HH) model is a system of one nonlinear diffusion equation for the membrane potential coupled with three ordinary differential equations, which shows how cells can produce propagating pulses in multicellular organisms. The HH model provided the first qualitative description of the formation of action potentials, which forms also a basis for models of excitable membrane behavior.

In the mid-1950s, people sought to reduce the four-variable HH model to a two-dimensional system of equations for which phase plane analysis applies. By combining the gating variables with slow kinetics, FitzHugh developed a simplified model which retains the essence of cubic nature of the more complicated HH model [7]. At about the same time, Nagumo and colleagues developed a similar model [12]. The two-variable fast-slow phase plane model has become known as the *FitzHugh-Nagumo equations*. This system has been used for qualitative description of generic excitable media in ecology, chemistry, and neuroscience.