GLOBAL EXISTENCE AND LONG TIME DYNAMICS OF A FOUR COMPARTMENT BRUSSELATOR TYPE SYSTEM

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Abstract. In this work we consider a four compartment Brusselator system. The reaction terms of this system are of non constant sign, thus components of the solution are not bounded apriori, and functional means to derive apriori bounds will fail. We prove global existence of classical solutions, via construction of an appropriate lyapunov functional. We also prove global existence of weak solutions, that facilitates the analysis of global attractors. Furthermore, due to the sign changing nonlinearities, the asymptotic sign condition is not satisfied, causing further difficulties in proving the existence of a global attractor. These difficulties are circumvented via the use of the lyapunov functional constructed, along with the use of the uniform gronwall lemma. We are able to prove the existence of an $(L^2(\Omega), H^2(\Omega))$ attractor for the system, improving previous results in the literature from [23]. The Hausdorff and fractal dimensions of the attractor are also shown to be finite. In particular we derive a new lower bound on the Hausdorff dimension of the global attractor. We use numerical simulations, as well as numerical attractor reconstruction methods via non linear time series analysis, to validate our theoretical results.

    Keywords: reaction diffusion system, global existence, global attractor, Lyapunov functional, attractor reconstruction, chaotic dynamics.

AMS (MOS) subject classification: 35K57, 35B40, 35B41, 35B65, 37B25

1   Introduction

The object of the current manuscript is to investigate global existence and the long time dynamics, of the following four compartment Brusselator type reaction diffusion system.