DHAGE ITERATION METHOD FOR NONLINEAR FIRST ORDER HYBRID DIFFERENTIAL EQUATIONS WITH A MIXED PERTURBATION OF THE SECOND TYPE

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Abstract. In this paper the authors prove algorithms for the existence and approximation of solutions for an initial and a periodic boundary value problem for nonlinear first order ordinary hybrid differential equations with a linear perturbation of the second type via the Dhage iteration method. Examples are furnished to illustrate the hypotheses and main results in this paper.

Keywords. Hybrid differential equation; Dhage iteration method; Hybrid fixed point theorem; Approximation theorem; perturbation of the second type.

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1 Introduction

Given a closed and bounded interval $J = [0, T]$, $T \in \mathbb{R}$, consider the initial and periodic boundary value problems for first order nonlinear hybrid differential equations (in short HDE),

$$\left. \begin{aligned} \frac{d}{dt} \left[\frac{x(t) - h(t, x(t))}{f(t, x(t))} \right] + \lambda \left[\frac{x(t) - h(t, x(t))}{f(t, x(t))} \right] = g(t, x(t)), \ \ t \in J, \\ x(0) = \alpha_0, \end{aligned} \right\} \quad (1)$$

and

$$\left. \begin{aligned} \frac{d}{dt} \left[\frac{x(t) - h(t, x(t))}{f(t, x(t))} \right] + \lambda \left[\frac{x(t) - h(t, x(t))}{f(t, x(t))} \right] = g(t, x(t)), \ \ t \in J, \\ x(0) = x(T), \end{aligned} \right\} \quad (2)$$