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A COMPARISON OF TWO HOMOGENIZATION PRINCIPLES FOR TARGET-ENVIRONMENT NETWORKS AND SYSTEMS ON PERIODIC GRAPHS

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Abstract. Network differential equations on extremely large periodic networks arise in many applications such as material sciences and nanotechnology. Due to the high complexity caused by the underlying microstructure, such problems are very challenging from both the analytical and the numerical perspective. Two-scale averaging techniques lead to averaged models on the global scale of the physical domain. Their solutions provide a characterization of the overall behavior of the system of network differential equations. Other approaches applied a notion of two-scale convergence for network functions in order to derive a homogenized model on the physical domain and a reference cell reflecting the periodic micro-topology. This study shows, that both representations are equivalent in the sense that the solution of the variational homogenized model can be identified with the solution of the averaged model. In this way, the result obtained by the purely formal averaging technique becomes fully justified in a mathematical rigorous way. In addition, the solution of the variational homogenized model can directly be derived from the solution of the averaged model that is obtained by standard PDE-solvers in nearly no time.

Keywords. Homogenization theory, two-scale convergence, two-scale transform, variational problems on graphs and networks, diffusion-reaction models, microstructures, periodic networks

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