GLOBAL BEHAVIOR OF SOLUTIONS FOR A DIFFERENCE EQUATION OF THIRD ORDER

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Abstract. In this paper we consider the third-order rational difference equation

$$y_{n+1} = \frac{y_{n-1} - y_{n-2}}{1 - ay_{n-2}}, \ n \in \mathbb{N}_0,$$

where parameter a is a nonzero real number and the initial values y_{-2}, y_{-1}, $y_0 \in \mathbb{R} \backslash \left\{ \frac{1}{a} \right\}$. We here determine both the forms and the global behavior of the solutions of the above equation. Also, we show that the solutions are associated with Padovan numbers which contribute to explain the global behavior of the solutions.

Keywords. Rational difference equation; periodic solution; global behavior; Padovan numbers.

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1   Introduction

Nonlinear difference equations have gained a great importance recently. Especially, in science and engineering different nonlinear phenomena are modeled by these equations. Furthermore, the results about these equations present prototypes towards the development of the basic theory of nonlinear difference equations. Many researchers have investigated the behavior of the solutions of nonlinear difference equations. For example, Tollu et al. [3] investigated the solutions of two special types of Riccati difference equations

$$x_{n+1} = \frac{1}{1 + x_n} \text{ and } y_{n+1} = \frac{1}{-1 + y_n}$$