Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **25 (2018)** 15-29
 http://www.watam.org

NONTRIVIAL PERIODIC SOLUTIONS FOR A CLASS OF SECOND-ORDER DELAY DIFFERENTIAL EQUATIONS

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Abstract. By using Morse theory and Garlerkin methods, some new results on the existence of the nontrivial periodic solutions to the system of delay differential equations

$$z''(t) = -f(z(t-\tau))$$

are obtained, where $\tau > 0$ is a constant and $f \in C(\mathbb{R}^N, \mathbb{R}^N)$.

MSC: 34K13; 34K18; 58E50

Keywords : Delay differential equation; Periodic solution; Morse Theory; Garlerkin methods.

1 Introduction

In this paper we study the existence of periodic solutions for the system of second order delay differential equations

$$z''(t) = -f(z(t-\tau)), \tag{1.1}$$

where $f : \mathbb{R}^N \mapsto \mathbb{R}^N$ is continuous and $\tau > 0$ is a given constant.

In the past several decades, the existence of periodic solutions for the functional differential equations has been extensively investigated via various approaches, including fixed point theorems, Hopf bifurcation theorem, Poincaré-Bendixson theorem and other effective methods (see e.g.[6, 7, 8, 11, 13, 16]).

Recently, many results on the existence of periodic solutions to delay differential equations are obtained via critical point theory [3, 4, 5, 9, 10, 18, 17]. Motivated by the work in [15, 18], in this paper, we study the existence of the nontrivial periodic solutions to the system (1.1) by using Morse theory and Garlerkin methods.