

ON THE SOLUTIONS OF A $(2K + 2)$ TH ORDER DIFFERENCE EQUATION

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Abstract. In this paper, we introduce an explicit formula and discuss the global behavior of solutions of the difference equation

$$x_{n+1} = \frac{ax_{n-2k-1}}{b - c \prod_{l=0}^k x_{n-2l-1}}, \quad n = 0, 1, \dots$$

where a, b, c , are non-negative real numbers, the initial conditions $x_{-2k-1}, x_{-2k}, \dots, x_{-1}, x_0$ are real numbers and k is a non-negative integer.

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1 Introduction

The higher-order non-linear difference equations are of paramount importance in applications. Such equations also seem naturally as discrete analogues and as numerical solutions of differential and delay differential equations which model various diverse phenomena in biology, ecology, physiology, physics, engineering, economics and so on. The theory of difference equations gets a central position in applicable analysis. That is, the theory of difference equations will continue to play an important role in mathematics as a whole. Hence, it is very valuable to investigate the qualitative behavior of difference equations. One way is to study rational difference equations via the exact expression of solutions (see [1, 2, 8, 13, 19, 20, 21, 23] and the references cited therein). Another way is studying the qualitative behavior such as asymptotical stability using the linearized method, semicycle analysis and so on (see [11, 12, 15, 18, 22] and the references cited therein). More results concerning with difference equations can be found in [6, 7, 10, 14, 16, 17].

In [4], the author studied the global behavior of the solutions of the difference equation

$$x_{n+1} = \frac{A + Bx_{n-1}}{C + Dx_n^2}, \quad n = 0, 1, \dots$$