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OSCILLATION AND ASYMPTOTIC BEHAVIOR OF ODD ORDER DELAY AND ADVANCED TYPE NEUTRAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we study the oscillation of odd order nonlinear neutral differential equation of the form

$$(x(t) + ax(t - \tau_1) + bx(t + \tau_2))^{(n)} + p(t)x^{\alpha}(t - \sigma_1) + q(t)x^{\beta}(t + \sigma_2) = 0, t \geq t_0 > 0,$$

where $n \geq 3$ is an odd integer, using arithmetic-geometric mean inequality. Examples are provided to illustrate the main results.

Keywords. Odd order, nonlinear neutral differential equation, oscillation, asymptotic behavior.

AMS subject classification: 34C10, 34K11.

1 Introduction

In this paper, we study the oscillation and asymptotic of odd order nonlinear neutral differential equation of the form

$$(x(t)+ax(t-\tau_1)+bx(t+\tau_2))^{(n)}+p(t)x^{\alpha}(t-\sigma_1)+q(t)x^{\beta}(t+\sigma_2) = 0, t \geq t_0 > 0, \tag{1.1}$$

where $n \geq 3$ is an odd integer, under the following conditions:

(C_1) $p(t)$ and $q(t)$ are continuous real valued functions on $[t_0, \infty)$;

(C_2) a, b, τ_1, τ_2, σ_1 and σ_2 are nonnegative constants;

(C_3) α and β are the ratios of odd positive integers.

By a solution of equation (1.1), we mean a continuous real valued function $x(t)$ on $[T_x, \infty)$, $T_x \geq t_0$, which is continuously n-times differentiable function on $[T_x, \infty)$ and satisfying the equation (1.1) for all $T_x \geq t_0$. We consider only those solutions $x(t)$ of equation (1.1) which satisfy $\sup\{|x(t)|; t \geq T\} > 0$ for all $T \geq T_x$. Also we assume that equation (1.1) possesses such solutions.

A nontrivial solution of a differential equation is said to be oscillatory if it has infinitely many zeros and nonoscillatory otherwise. A nontrivial solution of a differential equation is said to be almost oscillatory if it is either